

## A CHARACTERIZATION THEOREM IN TRIPLY DIFFUSIVE CONVECTION WITH VISCOSITY VARIATIONS

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**Abstract:** The paper mathematically establishes that triply diffusive convection, with variable viscosity and with one of the components as heat with diffusivity  $\kappa$ , cannot manifest itself as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the two concentration Rayleigh numbers  $R_1$  and  $R_2$ , the Lewis numbers  $\tau_1$  and  $\tau_2$  for the two concentrations with diffusivities  $\kappa_1$  and  $\kappa_2$  respectively (with no loss of generality  $\kappa > \kappa_1 > \kappa_2$ ),  $\mu_{\min}$  (the minimum value of viscosity  $\mu$  in the closed interval  $[0, 1]$ ) and the Prandtl number  $\sigma$  satisfy the inequality  $R_1 + R_2 \leq \frac{27\pi^4}{4} \left\{ \frac{\mu_{\min} + \frac{(\tau_1 + \tau_2)}{\sigma}}{1 + \frac{\tau_1}{\tau_2}} \right\}$  provided  $D^2\mu$  is positive everywhere. It is further proved that this result is uniformly valid for the quite general nature of the bounding surfaces.

**Key Words:** Triply diffusive convection, variable viscosity, concentration Rayleigh number, oscillatory motion, initially bottom heavy configuration.

**Mathematical Subject Classification Number:** 76E06, 76E20.

### 1. Introduction

Only the case of two component systems has been considered by researchers. However, it has been recognized later on [4, 8, 7, 16, 19] that there are many situations wherein more than two components are present. Examples of such multiple diffusive convection fluid systems include the solidification of molten alloys, Earth core, geothermally heated lakes, and magmas and their laboratory models and sea water etc. The presence of more than one salt in fluid mixtures is very often requested for describing natural phenomena such as underground water flow, contaminant transport, warming of the stratosphere, acid rain effects. Further a number of technologically important alloys such as nickel-based alloys [8] used in turbine blades and another high-strength applications, containing significant mass fraction of as many as seven metallic elements.

The recently established characterization theorem of Prakash et al. [10], which states that oscillatory motions (neutral or unstable) of growing amplitude cannot manifest in an initially bottom heavy thermohaline convection of Veronis type whenever the sum of the concentration Rayleigh numbers is less than a critical value, has brought a fresh outlook to the subject matter of triply diffusive convection and paved the way for further theoretical and experimental investigations in this field of enquiry. The summary of Prakash et al.'s [10] characterization theorem is that it provides a classification of the neutral and unstable triply diffusive convection classes namely the bottom heavy class and the top heavy class and strikes a distinction between them by means of characterization theorems which disallow the existence of oscillatory motions in the former class. For the field of applications of the Prakash et al.'s [10] theorem in flows that are of interest in certain fields like geophysics, oceanography, astrophysics etc. it is necessary to extend the classical analysis where in the fluid viscosity is a function of temperature and /or depth because the effects of viscosity variation play an important role in several physical situations in these fields [2, 5, 6, 15, 17]. Since the variation of viscosity of liquids with temperature is extremely rapid, the inclusion of variation effects certainly extends the domain of validity of the existing results in the literature.

The considerations of a temperature dependent viscosity on the pattern of density in the triply convection problems has the limitations that viscosity is a linear function of vertical coordinate which need not necessarily be so in a real physical situation. Therefore, in the governing equations of the problem, we consider viscosity as an arbitrary function of the vertical coordinate which is in accordance with the formulation regarding the role of viscosity in Rayleigh-Taylor instability problem. From the mathematical point of view the resulting differential equations have variable coefficients contrary to the case wherein viscosity is constant and therefore these more general problems introduce extra analytical complexities. In the present chapter we make an attempt to mathematically handle these more complex problems in the context of Prakash et al.'s [10] theorem and extend the domain of validity of the earlier results in the literature.

## 2. Formulation of the problem

Consider an infinite horizontal layer filled with a Boussinesq viscous fluid statically confined between two horizontal boundaries  $z = 0$  and  $z = d$ , maintained at constant temperatures  $T_0$  and  $T_1 (< T_0)$  and uniform solute concentrations  $S_{10}$ ,  $S_{20}$  and  $S_{11} (< S_{10})$ ,  $S_{21} (< S_{20})$  at the lower and upper boundaries respectively (see Fig.1). It is further assumed that cross diffusion effects may be neglected.

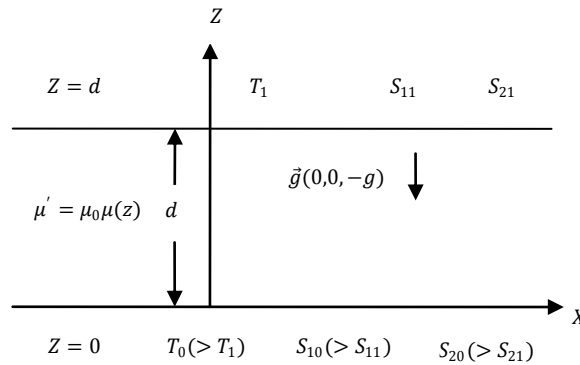


Fig.1 Physical Configuration

The governing hydrodynamic equations that govern the triply diffusive instability problem in non-dimensional form with variable viscosity are given by (Prakash [9], Prakash et al.[11])

$$\mu(D^2 - a^2)^2 w + D^2 \mu(D^2 + a^2)w + 2D\mu D(D^2 - a^2)w - \frac{p}{\sigma}(D^2 - a^2)w = Ra^2 \theta - R_1 a^2 \phi_1 - R_2 a^2 \phi_2, \tag{1}$$

$$(D^2 - a^2 - p)\theta = -w, \tag{2}$$

$$\left(D^2 - a^2 - \frac{p}{\tau_1}\right)\phi_1 = -\frac{w}{\tau_1}, \tag{3}$$

$$\text{and } \left(D^2 - a^2 - \frac{p}{\tau_2}\right)\phi_2 = -\frac{w}{\tau_2}, \tag{4}$$

with boundary conditions

$$w = 0 = \theta = \phi_1 = \phi_2 = D^2 w \text{ at } z = 0 \text{ and } z = 1 \tag{5}$$

(both boundaries are dynamically free)

$$\text{or } w = 0 = \theta = \phi_1 = \phi_2 = Dw \text{ at } z = 0 \text{ and } z = 1 \tag{6}$$

(both boundaries are rigid)

$$\text{or } w = 0 = \theta = \phi_1 = \phi_2 = Dw \text{ at } z = 0 \text{ (lower boundary is rigid)} \tag{7}$$

and  $w = 0 = \theta = \phi_1 = \phi_2 = D^2 w$  at  $z = 1$  (upper boundary is dynamically free)

$$\text{or } w = 0 = \theta = \phi_1 = \phi_2 = D^2 w \text{ at } z = 0 \text{ (lower boundary is dynamically free)} \tag{8}$$

and  $w = 0 = \theta = \phi_1 = \phi_2 = Dw$  at  $z = 1$  (upper boundary is rigid),

where  $z$  is the vertical coordinate,  $D = \frac{d}{dz}$  is the differentiation along the vertical direction,  $a^2 > 0$  is the square of the wave number,  $\sigma > 0$  is the Prandtl number,  $\tau_1 > 0$

and  $\tau_2 > 0$  are the Lewis numbers for the two concentration components respectively,  $R > 0$  is the Rayleigh number,  $R_1 > 0$  and  $R_2 > 0$  are the concentration Rayleigh numbers for the concentration components  $S_1$  and  $S_2$  respectively,  $p = p_r + ip_i$  is the complex growth rate,  $w$  is the vertical velocity,  $\theta$  is the temperature,  $\phi_1$  and  $\phi_2$  are the concentration of two components  $S_1$  and  $S_2$  respectively and  $\mu' = \mu_0\mu(z)$  is the viscosity, where  $\mu_0$  is constant having the dimensions of viscosity and  $\mu(z)$  is twice continuously differentiable function of  $z$  and is such that the ratio of the viscosities at the top and bottom boundaries is small [14]. It may further be noted that equations (1)-(8) describe an eigenvalue problem for  $p$  and govern triply diffusive convection with variable viscosity for any combination of dynamically free and rigid boundaries.

### 3. Mathematical Analysis

Now we prove the following theorem:

**Theorem:** If  $R > 0$ ,  $R_1 > 0$ ,  $R_2 > 0$ ,  $p_r \geq 0$ ,  $p_i \neq 0$ ,  $D^2\mu > 0$  and  $R_1 + R_2 \leq \frac{27\pi^4}{4} \left\{ \frac{\mu_{\min} + \frac{(\tau_1 + \tau_2)}{\sigma}}{1 + \frac{\tau_1}{\tau_2}} \right\}$  then a necessary condition for the existence of nontrivial solution  $(w, \theta, \phi_1, \phi_2, p)$  of equations (1)-(4) together with boundary conditions (5) or (6) or (7) or (8) is that  $R_s = R_1 + R_2 < R$ .

**Proof:** Equation (1) can further be simplified as

$$D(\mu D^3 w + D\mu D^2 w - 2a^2 \mu Dw) + a^4 \mu w - \frac{p}{\sigma} (D^2 - a^2)w + a^2 (D^2 \mu)w = Ra^2 \theta - R_1 a^2 \phi_1 - R_2 a^2 \phi_2. \quad (9)$$

Multiplying both sides of equation (9) by  $w^*$  (the superscript \* henceforth denotes complex conjugation), utilizing equations (2)-(4) and integrating the resulting equation over the vertical range of  $z$ , we get

$$\int_0^1 w^* D(\mu D^3 w + D\mu D^2 w - 2a^2 \mu Dw) dz + a^4 \int_0^1 \mu |w|^2 dz - \frac{p}{\sigma} \int_0^1 w^* (D^2 - a^2)w dz + a^2 \int_0^1 w^* (D^2 \mu)w dz = -Ra^2 \int_0^1 \theta (D^2 - a^2 - p^*) \theta^* dz + R_1 a^2 \tau_1 \int_0^1 \phi_1 (D^2 - a^2 - \frac{p^*}{\tau_1}) \phi_1^* dz + R_2 a^2 \tau_2 \int_0^1 \phi_2 (D^2 - a^2 - \frac{p^*}{\tau_2}) \phi_2^* dz. \quad (10)$$

Integrating the various terms of equation (10), by parts, for an appropriate number of times and using either of the boundary conditions (5)-(8), we get

$$\int_0^1 \mu (|D^2 w|^2 + 2a^2 |Dw|^2 + a^4 |w|^2) dz + \frac{p}{\sigma} \int_0^1 (|Dw|^2 + a^2 |w|^2) dz + a^2 \int_0^1 (D^2 \mu) |w|^2 dz = Ra^2 \int_0^1 (|D\theta|^2 + a^2 |\theta|^2 + p^* |\theta|^2) dz - R_1 a^2 \tau_1 \int_0^1 (|D\phi_1|^2 + a^2 |\phi_1|^2 + \frac{p^*}{\tau_1} |\phi_1|^2) dz - R_2 a^2 \tau_2 \int_0^1 (|D\phi_2|^2 + a^2 |\phi_2|^2 + \frac{p^*}{\tau_2} |\phi_2|^2) dz. \quad (11)$$

Equating the real and imaginary parts of both sides of equation (11) and cancelling  $p_i (\neq 0)$  throughout from the imaginary parts, we have

$$\int_0^1 \mu (|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2)dz + \frac{p_r}{\sigma} \int_0^1 (|Dw|^2 + a^2|w|^2) dz + a^2 \int_0^1 (D^2\mu) |w|^2 dz = Ra^2 \int_0^1 (|D\theta|^2 + a^2|\theta|^2 + p_r|\theta|^2) dz - R_1a^2\tau_1 \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2 + \frac{p_r}{\tau_1} |\phi_1|^2) dz - R_2a^2\tau_2 \int_0^1 (|D\phi_2|^2 + a^2|\phi_2|^2 + \frac{p_r}{\tau_2} |\phi_2|^2) dz. \tag{12}$$

$$\text{and } \frac{1}{\sigma} \int_0^1 (|Dw|^2 + a^2|w|^2) dz = -Ra^2 \int_0^1 |\theta|^2 dz + R_1a^2 \int_0^1 |\phi_1|^2 dz + R_2a^2 \int_0^1 |\phi_2|^2 dz. \tag{13}$$

Rearranging equation (12), we get

$$\begin{aligned} &\int_0^1 \mu (|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2)dz + \frac{p_r}{\sigma} \int_0^1 (|Dw|^2 + a^2|w|^2) dz + a^2 \int_0^1 (D^2\mu) |w|^2 dz \\ &= Ra^2 \int_0^1 (|D\theta|^2 + a^2|\theta|^2) dz - R_1a^2\tau_1 \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2) dz - R_2a^2\tau_2 \int_0^1 (|D\phi_2|^2 + a^2|\phi_2|^2) dz + p_r a^2 \left( R \int_0^1 |\theta|^2 dz - R_1 \int_0^1 |\phi_1|^2 dz - R_2 \int_0^1 |\phi_2|^2 dz \right). \end{aligned} \tag{14}$$

We derive the validity of the theorem from the inequality obtained by replacing each one of the terms of (14) by its appropriate estimate.

Utilizing Rayleigh –Ritz’s inequality [13], we have

$$\int_0^1 |Dw|^2 dz \geq \pi^2 \int_0^1 |w|^2 dz, \tag{15}$$

$$\int_0^1 |D\theta|^2 dz \geq \pi^2 \int_0^1 |\theta|^2 dz, \tag{16}$$

$$\int_0^1 |D\phi_1|^2 dz \geq \pi^2 \int_0^1 |\phi_1|^2 dz, \tag{17}$$

$$\text{and } \int_0^1 |D\phi_2|^2 dz \geq \pi^2 \int_0^1 |\phi_2|^2 dz. \tag{18}$$

$$\text{Further, we also have [1] } \int_0^1 |D^2w|^2 dz \geq \pi^4 \int_0^1 |w|^2 dz, \tag{19}$$

and thus making use of inequalities (15) and (19), we get

$$\int_0^1 \mu (|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2)dz \geq \mu_{\min} (\pi^2 + a^2)^2 \int_0^1 |w|^2 dz, \tag{20}$$

where  $\mu_{\min}$  is the minimum value of  $\mu$  in the closed interval  $[0, 1]$ .

$$\text{Since } p_r \geq 0, \text{ we get } \frac{p_r}{\sigma} \int_0^1 (|Dw|^2 + a^2|w|^2) dz \geq 0. \tag{21}$$

$$\text{Let } D^2\mu > 0, \text{ we have } a^2 \int_0^1 (D^2\mu) |w|^2 dz > 0. \tag{22}$$

Now multiplying equation (2) by  $\theta^*$ , integrating the resulting equation over the vertical range of  $z$  for an appropriate number of times and utilizing the boundary conditions on  $\theta$ , we have from the real part of final equation

$$\begin{aligned} \int_0^1 (|D\theta|^2 + a^2|\theta|^2) dz + p_r \int_0^1 |\theta|^2 dz &= \text{Real part of } \left( \int_0^1 \theta^* w dz \right) \leq \left| \int_0^1 \theta^* w dz \right| \\ &\leq \int_0^1 |\theta^* w| dz \leq \int_0^1 |\theta^*| |w| dz \leq \int_0^1 |\theta| |w| dz \leq \left( \int_0^1 |\theta|^2 dz \right)^{\frac{1}{2}} \left( \int_0^1 |w|^2 dz \right)^{\frac{1}{2}}. \end{aligned} \quad (23)$$

(using Schwartz Inequality)

Combining inequality (23) with inequality (16), and the fact that  $p_r \geq 0$ , we get

$$(\pi^2 + a^2) \int_0^1 |\theta|^2 dz \leq \left( \int_0^1 |\theta|^2 dz \right)^{\frac{1}{2}} \left( \int_0^1 |w|^2 dz \right)^{\frac{1}{2}},$$

$$\text{which gives that } \left( \int_0^1 |\theta|^2 dz \right)^{\frac{1}{2}} \leq \frac{1}{(\pi^2 + a^2)} \left( \int_0^1 |w|^2 dz \right)^{\frac{1}{2}}$$

$$\text{and hence } \int_0^1 (|D\theta|^2 + a^2|\theta|^2) dz \leq \frac{1}{(\pi^2 + a^2)} \left( \int_0^1 |w|^2 dz \right). \quad (24)$$

Further, utilizing inequalities (17) and (18) and equation (13), we get

$$\begin{aligned} R_1 a^2 \tau_1 \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2) dz + R_2 a^2 \tau_2 \int_0^1 (|D\phi_2|^2 + a^2|\phi_2|^2) dz \\ \geq (\pi^2 + a^2) \left( R_1 \tau_1 \int_0^1 |\phi_1|^2 dz + R_2 \tau_2 \int_0^1 |\phi_2|^2 dz \right) \\ \geq (\pi^2 + a^2) \left[ \frac{\tau_1}{\sigma} \int_0^1 (|Dw|^2 + a^2|w|^2) dz - R_2 a^2 \tau_1 \int_0^1 |\phi_2|^2 dz + \frac{\tau_2}{\sigma} \int_0^1 (|Dw|^2 + a^2|w|^2) dz - R_1 a^2 \tau_2 \int_0^1 |\phi_1|^2 dz \right]. \end{aligned} \quad (25)$$

Now multiplying equations (3) and (4) by their complex conjugate respectively, integrating the resulting equation over the vertical range of  $z$  for an appropriate number of times and utilizing the boundary conditions on  $\phi_1$  and  $\phi_2$ , we have

$$\begin{aligned} \int_0^1 (|D^2\phi_1|^2 + 2a^2|D\phi_1|^2 + a^4|\phi_1|^2) dz + \frac{2p_r}{\tau_1} \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2) dz + \\ \frac{|p|^2}{\tau_1^2} \int_0^1 |\phi_1|^2 dz = \frac{1}{\tau_1^2} \int_0^1 |w|^2 dz, \end{aligned} \quad (26)$$

$$\begin{aligned} \text{and } \int_0^1 (|D^2\phi_2|^2 + 2a^2|D\phi_2|^2 + a^4|\phi_2|^2) dz + \frac{2p_r}{\tau_2} \int_0^1 (|D\phi_2|^2 + a^2|\phi_2|^2) dz + \\ \frac{|p|^2}{\tau_2^2} \int_0^1 |\phi_2|^2 dz = \frac{1}{\tau_2^2} \int_0^1 |w|^2 dz. \end{aligned} \quad (27)$$

$$\text{We also have [10] } \int_0^1 |D^2\phi_1|^2 dz \geq \pi^4 \int_0^1 |\phi_1|^2 dz, \quad (28)$$

$$\text{and } \int_0^1 |D^2\phi_2|^2 dz \geq \pi^4 \int_0^1 |\phi_2|^2 dz. \quad (29)$$

Thus utilizing inequalities (17) and (28) for  $\phi_1$  and (18) and (29) for  $\phi_2$  in inequalities (26) and (27) respectively, and since  $p_r \geq 0$ , we obtain

$$\int_0^1 |\phi_1|^2 dz \leq \frac{1}{\tau_1^2 (\pi^2 + a^2)^2} \int_0^1 |w|^2 dz, \quad (30)$$

$$\text{and } \int_0^1 |\phi_2|^2 dz \leq \frac{1}{\tau_2^2(\pi^2+a^2)^2} \int_0^1 |w|^2 dz, \tag{31}$$

respectively.

Now using inequalities (30) and (31), we have

$$\begin{aligned} R_2 a^2 \tau_1 \int_0^1 |\phi_2|^2 dz + R_1 a^2 \tau_2 \int_0^1 |\phi_1|^2 dz &\leq \frac{a^2}{(\pi^2+a^2)^2} \left( \frac{R_2 \tau_1}{\tau_2^2} + \frac{R_1 \tau_2}{\tau_1^2} \right) \int_0^1 |w|^2 dz \\ &\leq \frac{a^2}{(\pi^2+a^2)^2} \left( \frac{R_2 \tau_1^3 + R_1 \tau_2^3}{\tau_1^2 \tau_2^2} \right) \int_0^1 |w|^2 dz. \end{aligned} \tag{32}$$

Using inequalities (15) and (32) in inequality (25),

$$\begin{aligned} R_1 a^2 \tau_1 \int_0^1 (|D\phi_1|^2 + a^2 |\phi_1|^2) dz + R_2 a^2 \tau_2 \int_0^1 (|D\phi_2|^2 + a^2 |\phi_2|^2) dz \\ \geq \left\{ (\pi^2 + a^2)^2 \frac{(\tau_1 + \tau_2)}{\sigma} - \frac{a^2}{(\pi^2+a^2)^2} \left( \frac{R_2 \tau_1^3 + R_1 \tau_2^3}{\tau_1^2 \tau_2^2} \right) \right\} \int_0^1 |w|^2 dz \end{aligned} \tag{33}$$

Let  $R_s = R_1 + R_2$ , thus  $R_2 = R_s - R_1$ .

Therefore  $R_2 \tau_1^3 + R_1 \tau_2^3 = R_s \tau_1^3 + R_1 (\tau_2^3 - \tau_1^3)$ .

Thus inequality (33) becomes

$$\begin{aligned} R_1 a^2 \tau_1 \int_0^1 (|D\phi_1|^2 + a^2 |\phi_1|^2) dz + R_2 a^2 \tau_2 \int_0^1 (|D\phi_2|^2 + a^2 |\phi_2|^2) dz \\ \geq \left\{ (\pi^2 + a^2)^2 \frac{(\tau_1 + \tau_2)}{\sigma} - \frac{a^2 R_s \tau_1}{(\pi^2+a^2)\tau_2^2} \right\} \int_0^1 |w|^2 dz \end{aligned} \text{ (since } \kappa_1 > \kappa_2, \text{ thus } \tau_1 > \tau_2 \text{)}$$

$$\begin{aligned} \text{and thus } -R_1 a^2 \tau_1 \int_0^1 (|D\phi_1|^2 + a^2 |\phi_1|^2) dz - R_2 a^2 \tau_2 \int_0^1 (|D\phi_2|^2 + a^2 |\phi_2|^2) dz \\ \leq \left\{ -(\pi^2 + a^2)^2 \frac{(\tau_1 + \tau_2)}{\sigma} + \frac{a^2 R_s \tau_1}{(\pi^2+a^2)\tau_2^2} \right\} \int_0^1 |w|^2 dz. \end{aligned} \tag{34}$$

Now from equation (13) and using  $p_r \geq 0$ , we get

$$p_r \left\{ R a^2 \int_0^1 |\theta|^2 dz - R_1 a^2 \int_0^1 |\phi_1|^2 dz - R_2 a^2 \int_0^1 |\phi_2|^2 dz \right\} < 0. \tag{35}$$

Now, if permissible, let  $R_s = R_1 + R_2 \geq R$ . Then, in that case, we derive from equation (14) by making use of inequalities (20)-(22), (24), (34) and (35) that

$$\left[ (\pi^2 + a^2)^2 \left\{ \mu_{\min} + \frac{(\tau_1 + \tau_2)}{\sigma} \right\} - \frac{R_s a^2}{(\pi^2+a^2)} \left\{ 1 + \frac{\tau_1}{\tau_2} \right\} \right] \int_0^1 |w|^2 dz < 0,$$

$$\text{which clearly implies that } R_s > \frac{(\pi^2+a^2)^3}{a^2} \left\{ \frac{\mu_{\min} + \frac{(\tau_1 + \tau_2)}{\sigma}}{1 + \frac{\tau_1}{\tau_2}} \right\},$$

$$\text{so that we necessarily have } R_s > \frac{27\pi^4}{4} \left\{ \frac{\mu_{\min} + \frac{(\tau_1 + \tau_2)}{\sigma}}{1 + \frac{\tau_1}{\tau_2}} \right\}, \tag{36}$$

since the minimum value of  $\frac{(\pi^2+a^2)^3}{a^2}$  is  $\frac{27\pi^4}{4}$  (for  $a^2 = \frac{\pi^2}{2}$ ).

Hence if  $R_s = R_1 + R_2 \leq \frac{27\pi^4}{4} \left\{ \frac{\mu_{\min} + \frac{(\tau_1+\tau_2)}{\sigma}}{1+\frac{\tau_1}{\tau_2}} \right\}$ , then we must have  $R_s = R_1 + R_2 < R$ .

This establishes the theorem.

The essential content of the theorem from the physical point of view are that for the problem of triply diffusive convection with variable viscosity, one of the components as heat with diffusivity  $\kappa$  cannot manifest as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the two concentration Rayleigh numbers  $R_1$  and  $R_2$ , the Lewis numbers  $\tau_1$  and  $\tau_2$  for the two concentrations with diffusivities  $\kappa_1$  and  $\kappa_2$  respectively (with no loss of generality  $\kappa > \kappa_1 > \kappa_2$ ),  $\mu_{\min}$  (the minimum value of  $\mu$  in the closed interval  $[0, 1]$ ) and the Prandtl number  $\sigma$  satisfy the inequality  $R_1 + R_2 \leq \frac{27\pi^4}{4} \left\{ \frac{\mu_{\min} + \frac{(\tau_1+\tau_2)}{\sigma}}{1+\frac{\tau_1}{\tau_2}} \right\}$  provided  $D^2\mu$  is positive everywhere. It is further proved that this result is uniformly valid for the quite general nature of the bounding surfaces.

**Note:** For the constant viscosity case it is clear that  $\mu' = \mu_0\mu(z)$  where  $\mu(z) = 1$  for all  $z$ ; therefore  $\mu_{\min}(z) = 1$  and we recover the result due to Prakash et al. [10] from the above theorem.

### Conclusion

Linear stability analysis of triply diffusive convection with variable viscosity has been performed. A classification of the neutral or unstable triply diffusive convection configuration with variable viscosity has been made into two classes namely, the bottom heavy class and top heavy class and then a distinction between these classes is made by means of a characterization theorem which disallow the existence of oscillatory motions in the former class. The work done in the present paper will certainly pave the way for further theoretical and experimental investigation in this field of enquiry.

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