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MACHINE REPAIR PROBLEM WITH SPARES, BALKING, RENEGING AND N-POLICY FOR VACATION

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Abstract: Present investigation deals with machine repair problem, balking, reneging and vacation. In this system, we consider two repairmen, the first repairman is always available for providing service to the failed units while the second repairman goes on vacation when the failed units are less than a threshold value (say N). After vacation the second repairman comes back when there are N or more failed units are present in the system. If the failed units are less than N, then the repairman goes on another vacation. Service and vacation times are exponentially distributed. We construct the steady state probabilities equations and apply the recursive technique to solve them.

Key Words: Vacation, repairable system, spares, balking, reneging.

1. Introduction

In this study we consider aN-policy for vacation policy of a machine repair problem with spares, balking and reneging. Such situation arises in production and manufacturing processes when an operating machine fails.An optimal repair policy for machine interference problem was developed by Elsayed[1]. Hsies and Wang [3]analysed the reliability of a repairable system with spares and removable repairman. Machine interface problem with warm spares, server vacation and exhaustive service was developed by Gupta [2]. Wang and Wu [9] studied cost analysis of the M/M/R machine repair problem with spares and two mode of failure. Chakraverty[14]analysed machine repair with an unreliable server and phase type repairs and services. Wang et al. [8]considered reliability and sensitivity analysis of a repairable system with warm standby and R unreliable service station. Jain and Upadhyaya[6]discussed threshold N-policy for degraded machining system with multiple types of spares and multiple vacation.

Wang [10] developed profit analysis of M/M/R machine repair problemwith spares and server breakdown. Jain et al. [7] and Jain and Singh [5] studied the machine repair problem with spears. A vacation policy for machine repair problem with two types of spares wasanalysedKee and Wang [11].

Kee and Wang [4] considered reliability analysis of balking and reneging in repairable system with warm standby. Probabilistic analysis of a repairable system with warm standby, balking and reneging was considered by Wang and Ke[15]. Jain and Sharma [12] studied controllable multi server queue with balking and additional server. Finite capacity queueing system with queue dependent server and discouragement was developed by Jain and Sharma [13].

In this study, we focus our attention on the machine repair problem with balking, reneging, spares and vacation as usually happens in real practical problem. The first repairman is always available for providing service, on the other hand the second repairman start service when there are N or more than N failed unit are accumulated. Rest of the paper is organized as follow. Section 2 provide the mathematical formulation by stating underline assumptions. The transient analysis of the model is given in section 3. Performance characteristics of the system are obtained in section 4. The cost analysis is given in section 5. In the final section 6, conclusions are drawn.

2. Model Description

Consider a finite capacity machine repair problem with spares, balking, reneging and vacation. In the system, we have “m” operating units and “s” spares with two repairmen. The first repairman always available for providing service to the failed units, while the second repairman turns on when there are N or more than N failed units are present in the system.If there are less than N failed units than second repairman goes on vacation of random length. After vacation, if the second repairman finds N or more than N failed units, it will start repair otherwise goes for a other vacation. The vacation time is exponentially distributed with rate "θ". When the operating unit is fails, it immediately replace by the spare unit if available and the fail unit send for repair. The failed unit is considered as good as new one after repair. These repaired units send to the operating side if there are less than m operating units. The system breakdown if and only if L=s+m-k+1 (m>k) or more units fail.

The failure characteristics of the spares are same as an operating machine. The life time of operating and spare machine are exponentially distributed with rate λ.

Joining probability is define as follow :

$$\beta_n = \begin{cases} \beta & 0 \le n \le N \\ \beta^{/} & N < n \le L \end{cases}$$

The repair times of both the repairman are exponentially distributed.

$$\mu_n = \begin{cases} \mu + (n-1)\alpha & 1 \le n \le \mathrm{N} \\ \mu_f + (n-1)\alpha & N \le n \le \mathrm{L} \end{cases}$$

$\mu_f$is the faster repair rate of second repairman. $\alpha$is the reneging parameter of the repairmen.

$\xi(t) = \begin{cases} 0 & \text{first repairman provide service and second repairman is on vacation.} \\ 1 & \text{both the repairman are provide service.} \end{cases}$

L(t) number of failed unit in the repair facility at time t.

$P_{n,i} = \lim_{t\to\infty} \Pr\{$n customers are in the system at time t and $\xi(t) = i\}$.

3. The Steady State Analysis

The steady state equations governing the model are giving as follows:

$$-\lambda_0 \beta P_{0,0} + \mu P_{1,0} = 0 \tag{1}$$

$$\lambda_{n-1}\beta P_{n-1,0} - \{\lambda_n \beta + \mu + (n-1)\alpha\} P_{n,0} + (\mu + n\alpha) P_{n+1,0} = 0;\ 1 \le n \le N-2 \tag{2}$$

$$\lambda_{N-2}\beta P_{N-2,0} - \{\lambda_{N-1}\beta + \mu + (N-2)\alpha\} P_{N-1,0} + \{(\mu + (N-1)\alpha)\} P_{N,0} + \{(\mu_f + (N-1)\alpha)\} P_{N,1} = 0 \tag{3}$$

$$\lambda_{n-1}\beta P_{n-1,0} - \{\lambda_n \beta + \mu + (n-1)\alpha + \theta\} P_{n,0} + (\mu + n\alpha + \theta) P_{n+1,0} = 0;\quad N \le n \le L-1 \tag{4}$$

$$-\lambda_{L-1}\beta P_{L-1,0} + \{\mu + (L-1)\alpha + \theta\} P_{L,0} = 0 \tag{5}$$

$$\theta P_{N,0} - \{\lambda_N \beta^{/} + \mu_f + (N-1)\alpha\} P_{N,1} + \{(\mu_f + N\alpha)\} P_{N+1,1} = 0 \tag{6}$$

$$\theta P_{n,0} + \lambda_{n-1}\beta^{/} P_{n-1,1} - \{\lambda_n \beta^{/} + \mu_f + (n-1)\alpha\} P_{n,1} + \{(\mu_f + n\alpha)\} P_{n+1,1} = 0;\quad N+1 \le n \le L-1 \tag{7}$$

$$\theta P_{L,0} + \lambda_{L-1}\beta^{/} P_{L-1,1} - \{\mu_f + (L-1)\alpha\} P_{L,1} = 0 \tag{8}$$

With the normalizing condition

$$\sum_{n=0}^{L} P_{n,0} + \sum_{n=N}^{L} P_{n,1} = 1 \tag{9}$$

And $\lambda_n = \begin{cases} (m+s-n)\lambda & n = 0,1,2,\dots,L-1 \\ 0 & n = L \end{cases}$ (10)

$$\phi_n = \begin{cases} \prod_{j=0}^{n-1} \dfrac{\lambda_j \beta^j}{(\mu + j\alpha)} & 1 < n \le N \\ \prod_{j=N}^{n} A_j \prod_{j=0}^{N-2} \dfrac{\lambda_j \beta^j}{(\mu + j\alpha)} & N < n \le L,\ \ j = N, N+1, \dots, L \end{cases}$$

From equation (1) and (2), we have

$$P_{n,0} = \emptyset_n P_{0,0};\ \ 1 \le n \le N-1 \tag{11}$$

From equation (5), we have

$$P_{L,0} = A_L P_{L-1,0} \tag{12}$$

where $A_{L=}\frac{\lambda_{L-1}\beta}{\mu+(L-1)\alpha+\theta}$

By equation (4) and (5)

$$P_{n,0}=A_nP_{n-1,0};\quad N\le n\le L-1 \tag{13}$$

where $A_n=\frac{\lambda_{n-1}\beta}{\left[\mu+(n-1)\alpha+\theta\left\{1+\sum_{i=n+1}^{L}\prod_{j=n+1}^{i}A_j\right\}\right]};\ N\le n\le L-1$

By equation (7) and (8)

$$P_{n,1}=\frac{\lambda_{n-1}\beta'}{\mu_f+(n-1)\alpha}P_{n-1}+\frac{\theta}{\mu_f+(n-1)\alpha}\sum_{i=N+1}^{L}P_{i,0};\ N+1\le n\le L \tag{14}$$

On putting n=N+1 in equation (14) and equation (6)

$$P_{N,1}=\frac{\theta}{(\mu_f+(N-1)\alpha)}\psi_nP_{0,0} \tag{15}$$

where $\psi_n=\sum_{i=N}^{L}\phi_i$, then

$$P_{n-1,1}=\frac{\theta}{(\mu_f+(n-2)\alpha)}\psi_nP_{0,0} \tag{16}$$

Substituting equation (11) and (16) in equation (14), we get

$$P_{n,1}=\frac{\theta}{(\mu_f+(n-1)\alpha)}\psi_nP_{0,0} \tag{17}$$

where$\psi_n=\left[\frac{\lambda_{n-1}\beta'}{\mu_f+(n-1)\alpha}\psi_{n-1}+\sum_{i=n}^{L}\phi_i\right];N+1\le n\le L$

By using normalizing condition

$\sum_{n=0}^{L}P_{n,0}+\sum_{n=N}^{L}P_{n,1}=1$, we get

$$P_{0,0}=\left[1+\sum_{n=1}^{L}\phi_n+\sum_{n=N}^{L}\left\{\frac{\theta}{(\mu_f+(n-1)\alpha)}\psi_n\right\}\right]^{-1} \tag{18}$$

4. Reliability Measures

1. The average numbers of failed units in the system is given by

$$L_f=\sum_{n=1}^{N-1}nP_{n,0}+\sum_{n=N}^{L}nP_{n,1}$$

$$=\left[\sum_{n=1}^{N-1}n\emptyset_n+\sum_{n=N}^{L}n\left\{\frac{\theta}{(\mu_f+(n-1)\alpha)}\psi_n\right\}\right]P_{0,0} \tag{19}$$

2. The average number of units that function as spears can be determine as

$$L_s = \begin{cases} \left[\sum_{n=N}^{s-1}(s-n)\emptyset_n + \sum_{n=N}^{s-1}(s-n)\left\{\frac{\theta}{(\mu_f+(n-1)\alpha)}\psi_n\right\}\right]P_{0,0} & N < s \\ \sum_{n=0}^{s-1}(s-n)\emptyset_n & N > s \end{cases} \quad ...(20)$$

3. The probability that first repairman is busy=Probability that there are no failed unit in the system

$$P_b^{(1)} = 1 - P_{0,0} \quad (21)$$

4. The probability that second repairman is busy = probability that there are N and more failed unit in the system

$$P_b^{(2)} = \sum_{n=N}^{L}\left\{\frac{\theta}{(\mu_f+(n-1)\alpha)}\psi_n\right\}P_{0,0} \quad (22)$$

5. Availability of the system is given by

$$A = 1 - \left(\emptyset_L + \frac{\theta}{(\mu_f+(L-1)\alpha)}\psi_L\right)P_{0,0}, \quad (23)$$

when L=s+m-k+1 (m>k) or more than L (system is not available).

6. Steady state failure frequency is obtained by

$$M = \lambda_{L-1}\left(\emptyset_{L-1} + \frac{\theta}{(\mu_f+(L-2)\alpha)}\psi_{L-1}\right)P_{0,0} \quad (24)$$

5. Cost Analysis

In this study, our main objective is to determine the optimum value of N i.e. N^*, so that we minimize the cost and availability of the system is maintained. For this purpose we use the following cost components to construct the cost function for proposed model are taken in to consideration.

C_f cost per unit time of one failed unit in the repair facility.

C_s cost per unit time of one unit that functions as a spare.

C_b^1 cost per unit time that repairman 1 is busy.

C_b^2 cost per unit time that repairman 2 is busy.

C_I cost per unit time that repairman 1 is idle.

C_v reward cost per unit time that the repairman 2 is on vacation.

$$E(N)=C_f.L_f + C_s.L_s + C_b^1.P_b^1 + C_b^2.P_b^2 + C_I P_I - C_v P_v \quad (25)$$

Where P_I(Probability that the first repairman is idle) = $P_{0,0}$

and P_v(Probability that the second repairman is on vacation) = $1-P_b^2$

maintain the availability of the system at a certain level, we have the cost minimization problem as follow:

$$\underset{s.t\ A\geq A_0}{Min} E(N)=C_f.L_f + C_s.L_s + C_b^1.P_b^1 + C_b^2.P_b^2 + C_I P_I - C_v P_v \quad (26)$$

Here A is the steady state availability of the system and A_0 is the given level of the availability of the system as a system parameter.

The optimal value (say N^*) of the decision variable N, could be determine by setting.

$$\frac{dE(N)}{dN} = 0. \quad (27)$$

In the case when N^* is not an integer, then the best positive integer value N^* is achieved by rounding off the N^*.

To evaluate the analytic results for the optimal value of N, which minimizing the expected total cost function would have been an arduous task to undertake as cost function is highly non linear and complex. For this purpose, heuristic approach method is use to obtain the optimum value N^* which is determined by satisfying the following inequalities:

$$E(N^*-1) > E(N^*) < E(N^*+1) \text{ and } A \geq A_0 \quad (28)$$

6. Discussion

We have investigated a machine repair problem with spares, balking, reneging and vacation of a repairman. We have provided the computational tractable formulae for the system characteristics and done cost analysis which may be useful to practitioners manufacturing and production processes. The present model has potential utility in many real time system where the machine are failed.

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