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HEAT TRANSFER ANALYSIS OF MHD VISCOUS FLOW OVER A SHRINKING SHEET IN PRESENCE OF VARIABLE THERMAL CONDUCTIVITY AND PARTIAL SLIP

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Abstract: The present work is concerned with the effects of variable thermal conductivity and heat source/sink on MHD viscous flow and heat transfer over a porous sheet which shrinks axisymmetrically in presence of partial slip. The slip is controlled by a dimensionless slip factor, which varies from zero to infinite and the thermal conductivity is assumed to vary linearly with the temperature. The governing equations are transformed to ordinary differential equations by using suitable similarity transformations and then solved numerically. Numerical results of velocity and temperature profiles are obtained with the effects of various parameters involved such as slip, suction, magnetic, thermal conductivity variation, Prandtl number and heat source/sink etc. and discussed them graphically in suitable manner such that interesting aspects of the solution can be adopted.

Key Words: Shrinking sheet, Slip condition, Thermal conductivity, Magnetic effect, Suction, Heat generation/absorption.

Introduction

The study of boundary layer flow over a stretching/shrinking sheet is a subject of great interest due to its various applications in manufacturing industries, technological process such as, plasma studies, petroleum industries, geothermal energy extractions, glass-fiber production, wire drawing, paper production, metal and polymer processing industries. In view of these applications, the boundary layer flow over a stretching sheet was first studied by Sakiadis [16]. Crane [9] found a closed form solution for steady two dimensional flows over a stretching sheet where the velocity on the boundary is proportional to the distance. The study of heat and mass transfer of the flow maintained at constant as well as variable wall temperature with suction was investigated by Gupta and Gupta [12]. Wang [19] studied the steady three dimensional flow of a viscous fluid over a plane surface which is stretched in its own plane in two lateral directions at different rates. Consequently, generalized three dimensional flow past a stretching sheet was considered by Ariel [4]. Miklavcic and Wang [14] investigated two-dimensional and axisymmetric viscous flow induced by a shrinking sheet in the presence of uniform suction. The above shrinking sheet problem was extended to power-law surface velocity by Fang [10]. An effect of heat and mass transfer on MHD boundary layer flow in the presence of suction was studied by Muhaimin [15]. Fang and Zhang [11] gave an exact solution of MHD boundary layer equations in closed analytical form for flow of an electrically conducting fluid. Bachok, et al. [6] extended the idea for unsteady three dimensional boundary flows due to a permeable shrinking sheet. Consequently, MHD viscous flow and heat transfer with prescribed surface heat flux was studied by Ali and Nazar [1].

A common feature of the above investigations and all the concerned references is the assumption that the flow field obeys the conventional no-slip condition at the sheet. In certain situations, however, the assumption of no-slip is not applicable and should be replaced by the partial slip boundary condition. Ariel [5] has studied the steady, laminar, axisymmetric flow of a Newtonian fluid due to a stretching sheet with a partial slip boundary condition. Further, Wang [19] has revived an interest in the viscous flow due to a stretching sheet with slip and suction. Boundary layer flow and heat transfer over a permeable shrinking sheet with partial slip was discussed by Aman and Ishak [2]. Recently, Bhattacharyya [7] gave slip effects on boundary layer stagnation-point flow and heat transfer towards a shrinking sheet. Also, all the above investigators restrict their analyses to viscous flow and heat transfer over a shrinking sheet with constant thermal conductivity. It was observed by Kays [13] that for liquid metals, the thermal conductivity varies linearly with temperature in the range 0-4000 F. With this assumption, Chaim [8] studied heat transfer in a fluid with variable thermal conductivity over stretching sheet and then the effects of variable thermal conductivity, viscous dissipation on steady MHD natural convection flow of low Prandtl fluid on an inclined porous plate with Ohmic heating was given by Sharma and Singh [17]. Radiative flow with variable thermal conductivity over a non isothermal stretching sheet in a porous medium was investigated by Vyas and Rai [20]. Hence we assume that the thermal conductivity is a linear function of the temperature.

The object of present paper is to study the effect variable thermal conductivity on three dimensional MHD viscous flow and heat transfer of an electrically conducting fluid past a porous axisymmetric permeable shrinking sheet with partial slip boundary condition.

Formation of the Problem

Consider a MHD viscous flow of an incompressible electrically conducting fluid due to a porous axisymmetric shrinking sheet. The sheet is placed in the plane $z=0$ and the flow takes place in upper half plane $z>0$. A constant magnetic field with strength B_0 is applied in the z direction (Fig. 1). The magnetic Reynolds number is taken to be small, so that the induced magnetic field is neglected. The flow is caused by the sheet which shrinks axisymmetrically in linear fashion. A uniform suction is applied normal to sheet to contain the vorticity and the thermal conductivity of the fluid is assumed to vary linearly with the temperature. The fluid adheres to the sheet partially and thus the motion of the fluid exhibits the slip condition. All the other fluid properties are assumed to be constant throughout the motion.

Under the usual boundary layer approximations, the basic governing boundary layer equations with heat source/sink are given as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial z^2}\right)-\frac{\sigma B_0{}^2}{\rho}u \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial z^2}\right)-\frac{\sigma B_0{}^2}{\rho}v \tag{3}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w}{\partial z^2}\right) \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{1}{\rho c_p}\frac{\partial}{\partial z}\left(\kappa^*(T)\frac{\partial T}{\partial z}\right)+\frac{Q\left(T-T_\infty\right)}{\rho c_p} \tag{5}$$

where (u,v,w) be the velocity components along the (x,y,z) directions, respectively, T is the temperature, T_∞ is the free stream temperature, p is the pressure, ρ is the density of the fluid, μ is the dynamic viscosity, $\nu=\frac{\mu}{\rho}$ is the kinematic viscosity, σ is the electrical conductivity, B_0 is the magnetic induction, κ^* is the variable thermal conductivity, c_p is the specific heat at constant pressure and Q is the volumetric rate of heat generation or absorption.

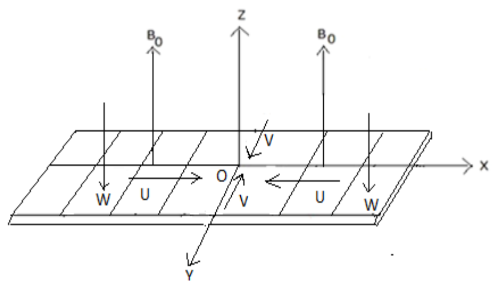

Fig. 1: Systematic diagram of physical model

The boundary conditions applicable to the present flow are

$$z = 0: \quad u = -U + \lambda_0 \left(\frac{\partial u}{\partial z} \right), \quad v = -V + \lambda_0 \left(\frac{\partial v}{\partial z} \right), \quad w = -W \text{ and } T = T_w$$

$$z \to \infty: \quad u \to 0,\ v \to 0 \text{ and } T \to T_\infty \tag{6}$$

where $U = ax$ and $V = ay$ are the shrinking velocities, $a > 0$ is the shrinking constant, $W > 0$ is the suction velocity, λ_0 is the slip coefficient, T_w is the sheet temperature and T_∞ is the free stream temperature.

Analysis

Introducing the following similarity transformations

$$u = axf'(\eta), \quad v = ayf'(\eta), \quad w = -2\sqrt{a\nu}\, f(\eta),$$

$$\eta = \sqrt{\frac{a}{\nu}}\, z \text{ and } \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{7}$$

In order to obtain the similarity solutions, it is assumed that the variable thermal conductivity κ^* of the fluid is varying linearly with temperature i.e. $\kappa^* = \kappa\left(1 + \in \theta\right)$, where κ is the thermal conductivity of the fluid and $\in$ is the thermal conductivity variation parameter.

Equation (1) is satisfied by similarity transformations while equation (4) can be integrated to give

$$\frac{p}{\rho} = \nu \frac{\partial w}{\partial z} - \frac{w^2}{2} + \text{Constant} \tag{8}$$

Equations (2) and (3) reduce to equation (9) and equation (5) reduces to equation (10) as

$$f''' + 2ff'' - f'^2 - (M + \frac{1}{K}) f' = 0$$

$$\left(1 + \in \theta\right)\theta'' + \in \theta'^2 + Pr\left[2f\theta' + B\theta\right] = 0 \tag{10}$$

Corresponding boundary conditions are

$$\eta = 0: \quad f = S, \quad f' = -1 + \lambda f'' \text{ and } \theta = 1$$

$$\eta \to \infty: \quad f' \to 0 \text{ and } \theta \to 0$$

where a prime denotes differentiation with respect to similarity variable η, $S=\frac{W}{2\sqrt{a\nu}}$ is the Suction parameter, $\lambda=\lambda_0\sqrt{\frac{a}{\nu}}$ is the Slip parameter, $M=\frac{\sigma B_0{}^2}{\rho a}$ is the Magnetic parameter, $Pr=\frac{\mu c_p}{\kappa}$ is the Prandtl number, $B=\frac{Q}{a\rho c_p}$ is the Heat Source $(B>0)$ or Sink $(B<0)$ parameter.

The physical quantities of interest are the local skin friction coefficient C_f on the surface along the x and y directions, which are denoted by C_{fx} and C_{fy}, respectively, and the local Nusselt number Nu i.e. surface heat transfer are given by

$$C_{fx}=\frac{\tau_{wx}}{\rho U^2/2}=\frac{\mu\left(\frac{\partial u}{\partial z}\right)_{z=0}}{\rho U^2/2}=\frac{2}{\sqrt{\mathrm{Re}_x}}f''(0),$$

$$C_{fy}=\frac{\tau_{wy}}{\rho V^2/2}=\frac{\mu\left(\frac{\partial v}{\partial z}\right)_{z=0}}{\rho V^2/2}=\frac{2}{\sqrt{\mathrm{Re}_y}}f''(0) \text{ and}$$

$$Nu=-\frac{x\left(\frac{\partial T}{\partial z}\right)_{z=0}}{\left(T_w-T_\infty\right)}=-\sqrt{\mathrm{Re}_x}\,\theta'(0) \tag{12}$$

where τ_{wx} and τ_{wy} are the wall shear stresses along the x and y directions, respectively and $\mathrm{Re}_x=\frac{ax^2}{\nu}$ and $\mathrm{Re}_y=\frac{ay^2}{\nu}$ are the local Reynolds numbers.

Results and Discussion

The set of nonlinear ordinary differential equations (9) and (10) with boundary conditions (11) are solved numerically using Runge - Kutta forth order algorithm with a systematic guessing of $f''(0)$ and $\theta'(0)$ by the shooting technique until the boundary conditions at infinity satisfied. The step size $\Delta\eta=0.001$ is used while obtaining the numerical solution and accuracy up to the seventh decimal place i.e. 1×10^{-7} which is very sufficient for convergence. In this method, we choose suitable finite values of $\eta\to\infty$, say η_∞, which depend on the values of the parameters used. All the computations were done by a program which uses a symbolic and computational computer language Matlab.

Velocity profiles for various values of slip parameter λ, magnetic parameter M and suction parameter S are shown in Figures 2, 3and 4, respectively. From all these figures it is observed that the velocity of the fluid at a point increases for increasing values of slip parameter, magnetic parameter and also for suction parameter.

The skin-friction coefficient $f''(0)$ is plotted against the slip parameter λ for different values S and M Figures 5 and 6, respectively. It is found that for increasing of S and M the skin friction coefficient $f''(0)$ increases, while for increasing values of slip parameter the $f''(0)$ decreases and approaches to zero, i.e., the fluid behaves like it is inviscid.

The temperature profiles for various parameters are given in figures 7 to 11. From the figures 7 and 8, it is noticed that the temperature profile and the thermal boundary layer thickness decreases for increasing values of slip and suction parameters. Figure 9 exhibits that as the variable thermal conductivity parameter ε increases the temperature profile also increases, which in turn increases the thermal boundary layer thickness as well. It is predicted from figure 10 that the temperature of fluid as well as thermal boundary layer thickness decreases with increase of Prandtl number. This happens because when Pr increases, the thermal conductivity decrease, thus it leads to the decrease of the energy transfer ability. From figure 11 it is found that the thickness of the thermal boundary layer reduces for increasing strength of heat sink $B<0$ but it increases for increasing values of heat source parameter.

Analysis for the temperature gradient at the sheet $-\theta'(0)$ for different values of parameters is tabulated in Table-1. It can be seen that for the prescribed range of parameters, the values of $-\theta'(0)$ are all positive. Physically, this indicates that the heat flow is always transferred from the surface of the sheet to the fluid. From the table, it is clear that in the presence of the slip, the rate of heat transfer $-\theta'(0)$ increases from the sheet to the fluid. It is also evident that for increasing values of B and $\in$, the heat transfer rate decreases but it increases with increasing of Pr and S.

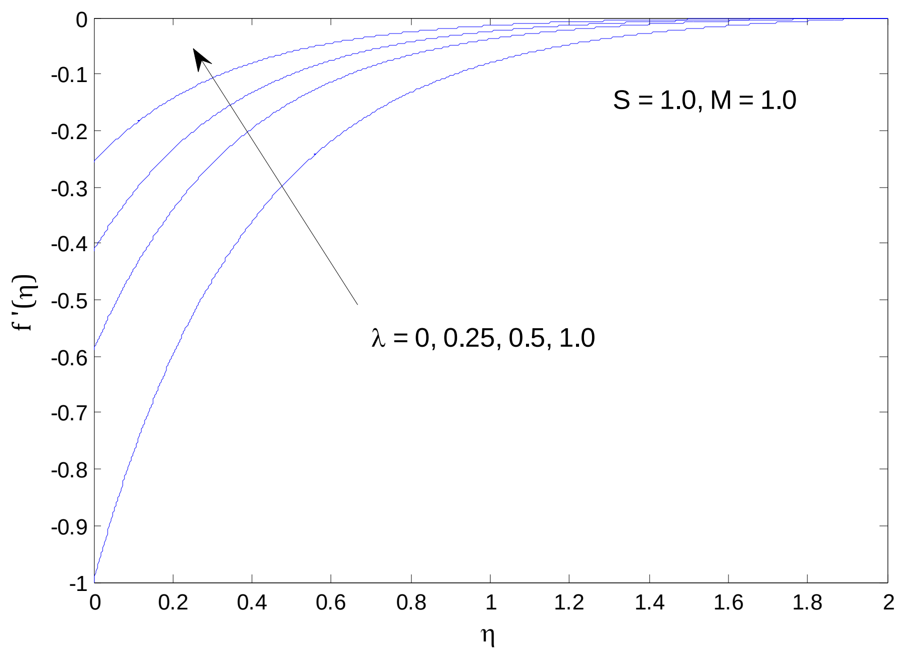

Fig.2: Velocity profile for various values of slip parameter λ .

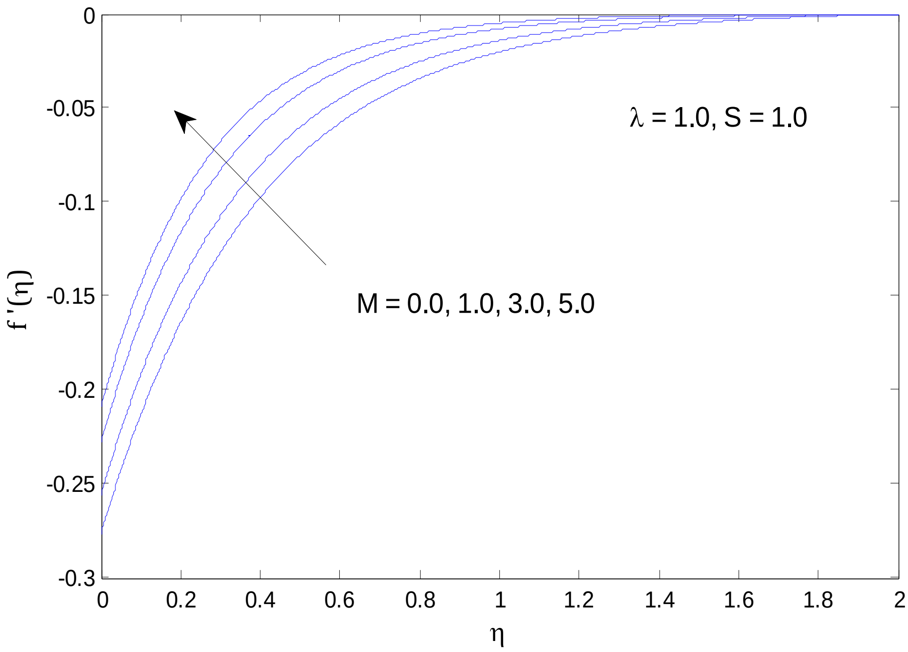

Fig.3: Velocity profile for various values of magnetic parameter M .

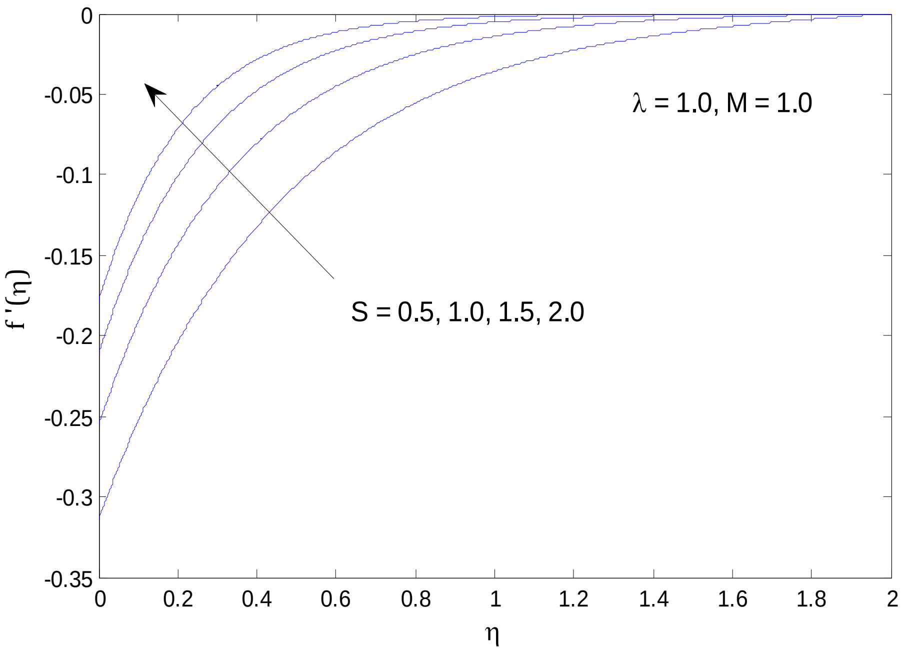

Fig.4: Velocity profile for various values of suction parameter S .

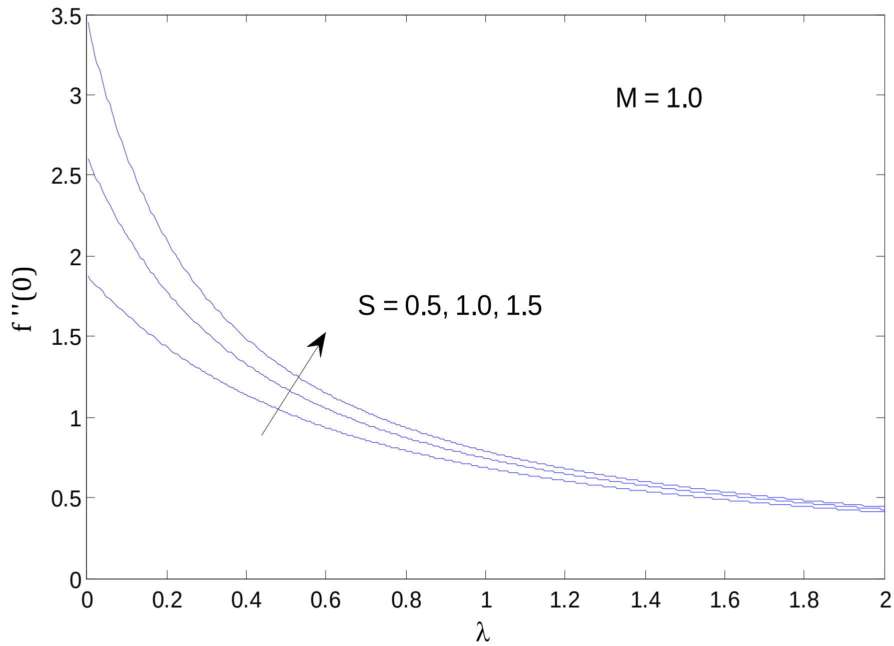

Fig.5: Skin friction coefficient against λ for various values of suction parameter S .

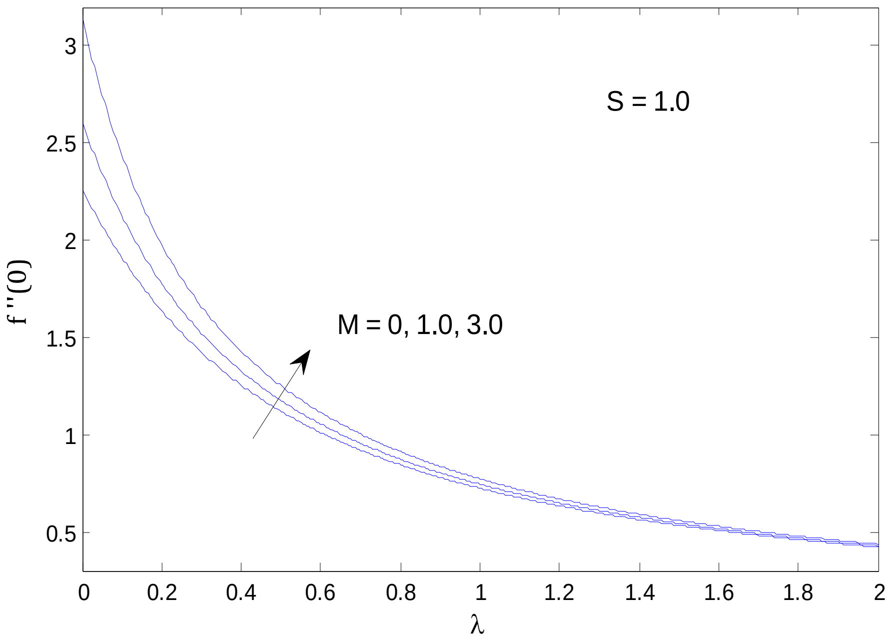

Fig.6: Skin friction coefficient against λ for various values of magnetic parameter M .

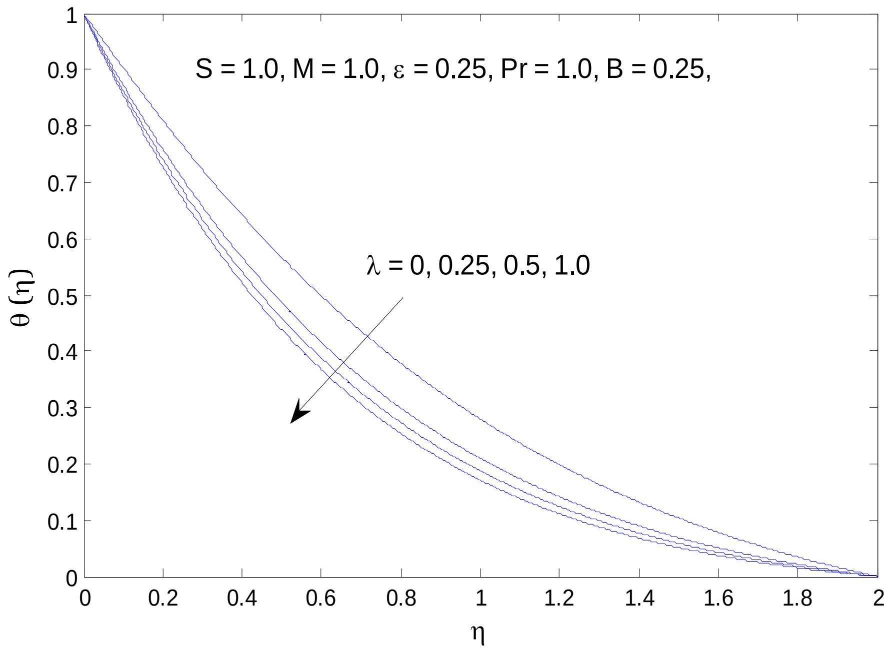

Fig.7: Temperature profile against η for various values of slip parameter λ .

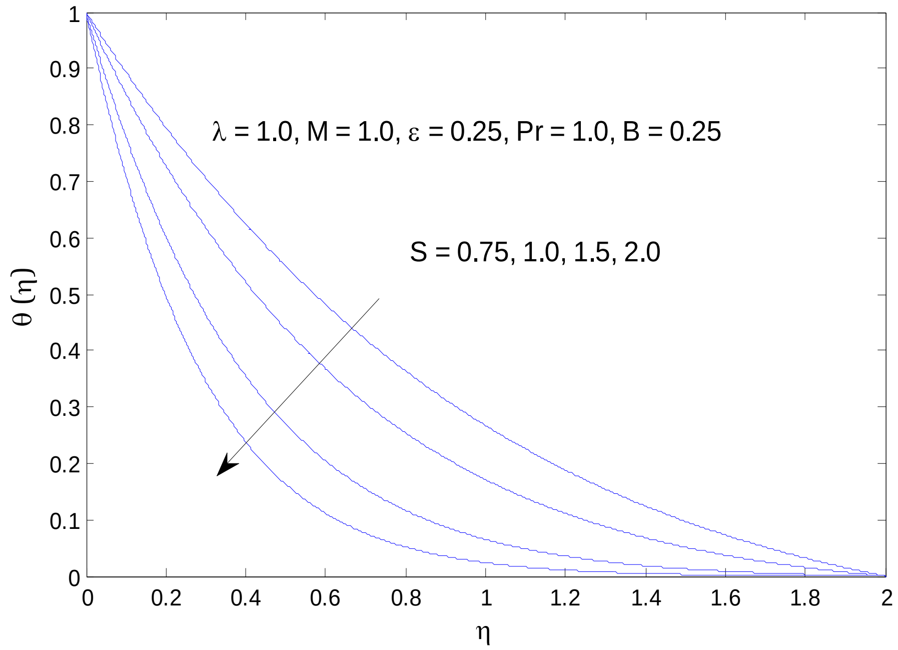

Fig.8: Temperature profile for various values of suction parameter S .

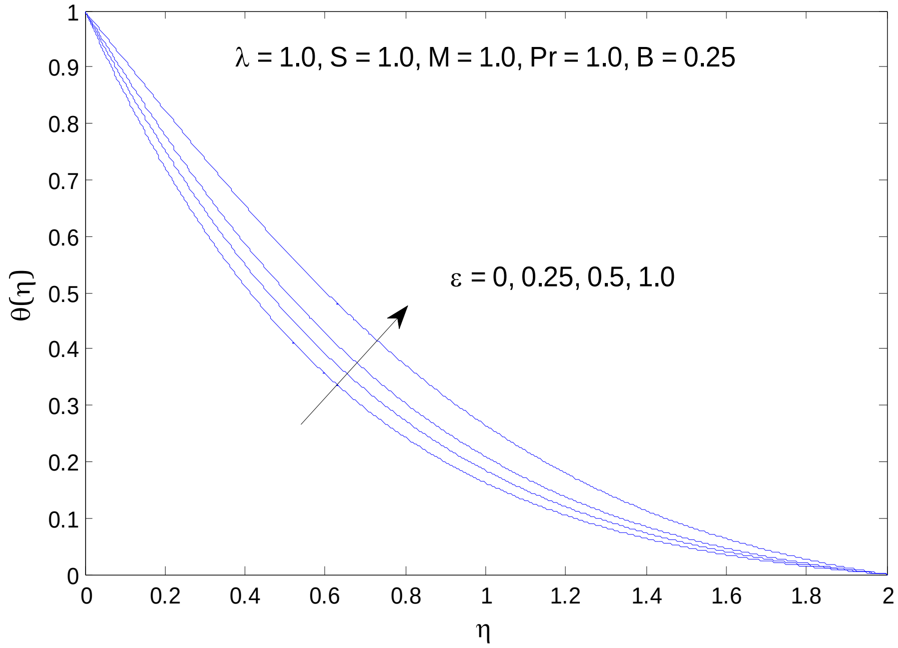

Fig.9: Temperature profile for various values of thermal conductivity parameter $\in$.

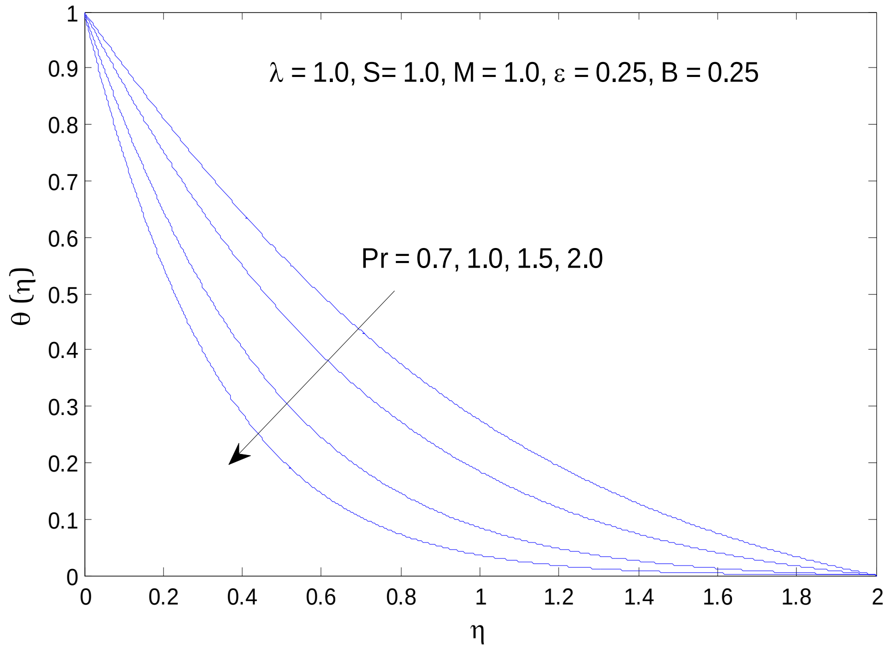

Fig.10: Temperature profile for various values of Prandtl number *Pr* .

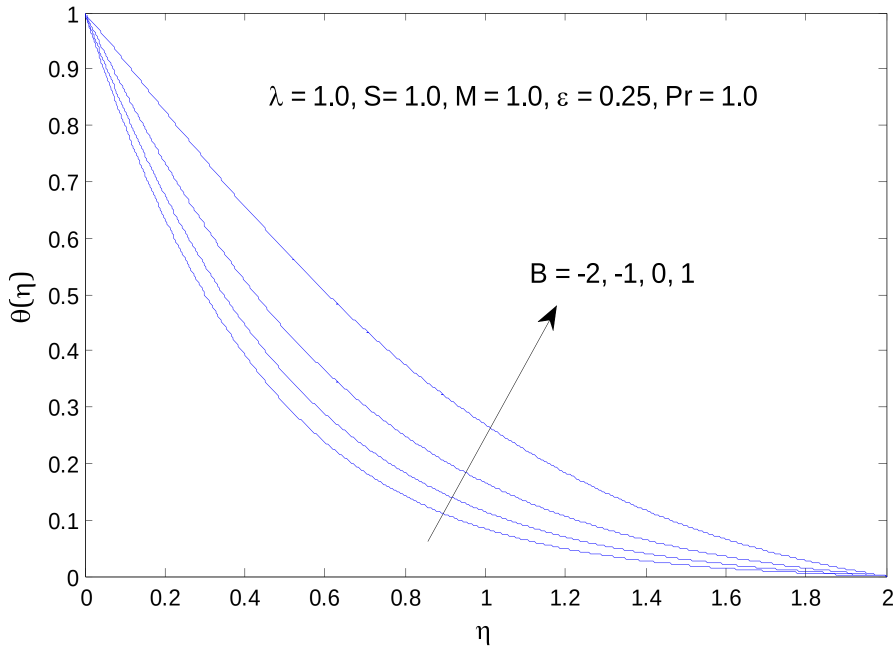

Fig.11: Temperature profile for various values of heat source/sink parameter *B* .

Table 1: Values of $-\theta'(0)$ for different values of the parameters

S	M	$\in$	Pr	$\lambda=0$, $B=-0.5$	$\lambda=0$, $B=0.5$	$\lambda=1$, $B=-0.5$	$\lambda=1$, $B=0.5$
1.0	1.0	0.25	0.7	1.05970	0.61522	1.39899	1.02977
			1.0	1.38826	0.78937	1.87680	1.42678
		0.5	0.7	0.93740	0.50979	1.24598	0.89353
			1.0	1.22469	0.64723	1.66763	1.23842
	3.0	0.25	0.7	1.00894	0.57387	1.40781	1.04221
			1.0	1.31483	0.73603	1.88886	1.44521
		0.5	0.7	0.88649	0.46703	1.25360	0.90461
			1.0	1.15162	0.59209	1.67808	1.25494
1.5	1.0	0.25	0.7	1.56157	1.21694	1.94252	1.65034
			1.0	2.16835	1.77217	2.69892	2.38058
		0.5	0.7	1.37231	1.04186	1.72178	1.44353
			1.0	1.90369	1.52389	2.38854	2.08566
	3.0	0.25	0.7	1.51304	1.17239	1.94734	1.65659
			1.0	2.09672	1.70605	2.70500	2.38853
		0.5	0.7	1.32452	0.99744	1.72597	1.44908
			1.0	1.83398	1.45877	2.39383	2.09273

Conclusions

In the present work the applicability of slip boundary conditions in boundary layer theory is explored and numerical similarity solutions are presented. The set of nonlinear ordinary differential equations with slip boundary conditions are solved numerically using Runge -Kutta forth order algorithm with the shooting technique. It is found that the slip decreases the velocity and the thermal boundary layer thickness. It is observed that the skin friction cofficient increases for increasing values of suction parameter and magnetic parameter. Moreover, it is observed that the skin friction cofficient decreases and approaches to zero for higher values of slip parameter, i.e., the fluid behaves as though it is inviscid. The rate of heat transfer at the sheet i.e. $-\theta'(0)$ decreases for increasing values of heat source/sink parameter and thermal conductivity variation parameter, while it increases with an increase of suction and Prandtl number.

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References

[1] Ali, F.M., Nazar, R. and Arifin, N.M. (2010). MHD viscous flow and heat transfer induced by permeable shrinking sheet with prescribed surface heat flux, *WSEAS Transaction on Mathematics*, 9, 365-375.

[2] Aman, F. and Ishak, A. (2010). Boundary layer flow and heat transfer over a permeable shrinking sheet with partial slip, *Journal of Applied Sciences Research*, 6(8) 1111-1115.

[3] Andersson, H.I. (2002). Slip flow past a stretching surface, *Acta Mech.* 158(1-2), 121–125.

[4] Ariel, P.D. (2003). Generalized three dimensional flow past a stretching sheet with suction, *ZAMM*, 83(12), 844-852.

[5] Ariel, P.D. (2007). Axisymmetric flow due to a stretching sheet with partial slip. *Comput. Math. Appl*., 54(7-8), 1169–1183.

[6] Bachok, N., Ishak, A. and Pop, I. (2010). Unsteady three-dimensional boundary layer flow due to a permeable shrinking sheet, *Applied Mathematics and Mechanics*, 31(11), 1421–1428.

[7] Bhattacharyya, K., Mukhopadhyay, S. and Layek, G.C. (2011). Slip effects on boundary layer stagnation-point flow and heat transfer towards a shrinking sheet, *International Journal of Heat and Mass Transfer*, 54, 308–313.

[8] Chaim, T.C. (1998). Heat transfer in a fluid with variable thermal conductivity over stretching sheet, *Acta Mechanica*, 129, 63-72.

[9] Crane, L.J. (1970). Flow past a stretching sheet, Z. Angew. Math. Phys, 21(4), 645–647.

[10] Fang, T. (2008). Boundary layer flow over a shrinking sheet with power-law velocity, *International Journal of Heat and Mass Transfer*, 51(25-26), 5838-5843.

[11] Fang, T. and Zhang, J. (2005). Closed-form exact solutions of MHD viscous flow over a shrinking sheet, *Communication in Nonlinear Science and Numerical Simulation*, 14, 2853-2857.

[12] Gupta, P.S. and Gupta, A.S. (1977). Heat and mass transfer on a stretching sheet with suction and blowing, *Canadian Journal of Chemical Engineering*, 55, 744-746.

[13] Kays, W.M. (1966). Convective heat and mass transfer, *McGraw-Hill, New York*, 362.

[14] Miklavcic, M. and Wang, C.Y. (2006). Viscous flow due to a shrinking sheet, *Quarterly Applied Mathematics*, 64, 283-290.

[15] Muhaimin, Kandasamy R. and Khamis, A.B. (2008). Effects of heat and mass transfer on nonlinear MHD boundary layer flow over a shrinking sheet in the presence of suction, *Applied Mathematics and Mechanics English Edition*, 29(10), 1309–1317.

[16] Sakiadis, B.C. (1960). Boundary-layer behavior on continuous solid surfaces: I. Boundary-layer equations for two-dimensional and axisymmetric flow, *AIChE Journal*, 7(1), 26–28.

[17] Sharma, P.R. and Singh, G. (2010). Effects of variable thermal conductivity, viscous dissipation on steady MHD natural convection flow of low Prandtl fluid on an inclined porous plate with Ohmic heating, *Meccanica*, 45, 237–247.

[18] Wang, C.Y. (1984). The three-dimensional flow due to a stretching at surface, *Phys. Fluids*, 27(8) 1915–1917.

[19] Wang, C.Y. (2009). Analysis of viscous flow due to a stretching sheet with surface slip and suction, *Nonl. Anal. Real World Appl*. 10(1), 375–380.

[20] Vyas, P. and Rai, A. (2010). Radiative flow with variable thermal conductivity over a non isothermal stretching sheet in a porous medium, *Journal of Contemporary Mathematical Sciences*, 5, 2685–2698.