

LRS BIANCHI-I DUST FILLED MODEL IN THE PRESENCE OF ZERO MASS SCALAR FIELDS WITH VARIABLE G AND Λ TERM

VIMAL CHAND JAIN

Department of Mathematics, Govt. Engineering College, Barliya Chouraha,
NH-8, Ajmer-305002 (India)
Email : vcjeca@gmail.com

NIKHIL JAIN

Department of Mathematics, Govt. Women Engineering College, Nasirabad
Road, Ajmer-305002 (India)
Email: jain10480@gmail.com

Received : June. 25, 2014

Abstract: LRS Bianchi type I dust filled cosmological model in the presence of zero mass scalar fields with variable gravitational and cosmological constants is investigated. To get the deterministic solution, we have assumed that shear (σ) is proportional to the expansion (θ), which leads to $A = \alpha B^n$ where A and B are metric potentials and α, n are real constants. In the present study, effect of zero mass scalar fields is discussed. It is observed that shear (σ), energy density (ρ) and expansion (θ) increase rapidly as cosmic time tends to zero and tends to zero as time increases indefinitely. The cosmological constant Λ is found to be positive and is a decreasing function of time, which is supported by results from recent Supernovae observations. Some physical and geometrical features of the model are also investigated.

Keywords: LRS-Bianchi Type-I model. Zero mass scalar fields. Varying G and Λ term.

2010 Mathematics Subject Classification : 83xx, 83Fxx, 83F05

1. Introduction

In recent years there has been a considerable interest in cosmological models of the universe which are important in understanding the mysteries of the early stage of its evolution. Several modifications of Einstein's general theory of relativity have been proposed and extensively studied so far by many cosmologists to unify gravitation and many other effects in the universe. The large scale of distribution of galaxies in our universe shows that the matter is satisfactorily described by a perfect fluid.

Many researchers have presented different forms of Bianchi type I models under various conditions. LRS Bianchi type I string dust magnetized cosmological models have been studied by Bali and Upadhaya [1]. Mazumdar [13] has obtained solution of an LRS Bianchi I space-time filled with a perfect fluid. Hajj-Boutros and Sfeila [8] and Sri Ram [29] also obtained some solutions for same field equations by using their solution generating techniques.

The importance of scalar fields (mesons) in cosmology has become well known. The study of interacting fields, one of the fields being a zero mass scalar field is basically an attempt to look into the yet unsolved problem of the unification of gravitational and quantum theories. In the last few decades there has been a lot of interest focused on the theory of gravitation representing zero mass scalar fields coupled with gravitational field. Pradhan et al. [19] have presented LRS Bianchi I cosmological model in the presence of zero mass scalar fields. Brahmchary [2], Gautreau [7] are some of researchers who have investigated various aspects of interacting fields. Patel [14] and Reddy [21] have investigated plane symmetric solutions of the field equations corresponding to zero mass scalar fields.

Bianchi type I anisotropic and homogeneous models are simple and are generally considered to portray early universe reasonably well. Another Bianchi type I model has been presented by Banerjee et al. [2] which obeys Takabayashi's equation of state. Kalligas et al. [11], Rehman [22] investigated cosmological models with variable G and

Λ using the condition $\rho \propto t^n$ and $G \propto t^m$ where m and n are constants, ρ is the energy density and G the gravitational constant.

The cosmological constant Λ and the gravitational constant G are the two parameters present in Einstein's field equations. The Newtonian constant of gravitation (G) plays the role of coupling constant between geometry and matter in Einstein's field equations. Dirac [4] was the first researcher, to propose the idea of a variable G on certain physical grounds. In the last few decades there have been numerous modifications of general relativity in which gravitational constant G varies with time. Singh and Kotambkar [27] have presented cosmological models with G and Λ in higher dimensional space time. Singh and Sorokhaibam [28] have studied FRW cosmological models with the gravitational and cosmological constants. Recently, Singh and Kale [26] have discussed anisotropic bulk viscous cosmological models with variable G and Λ .

Recent cosmological observations by High-Z Supernova Team and Supernova Cosmological Project (Garnavich et al. [6]; Schmidt et al. [25]) suggest the existence of a positive cosmological constant Λ with magnitude Λ (Gh/c^3) $\approx 10^{-123}$. Some of the recent discussions on the cosmological constant "problem" and the consequences on cosmology with a time varying cosmological constant have been discussed by Dologov[5]; Sahni and Starobinsky [24]; Peebles[15]; Pradhan et al. [17,18]; Jain [10] and Yadav et al.[30]. Recently Jain and Yadav [9] studied LRS Bianchi type II disordered radiation model with a varying Λ term.

This motivates us to study the Bianchi type cosmological models with zero- mass scalar fields in which G and Λ varies with time. In this paper we have investigated LRS Bianchi type I dust filled cosmological model with zero- mass scalar fields with variable G and Λ using the condition that shear (σ) is proportional to the scalar of expansion (θ) which leads to $A = \alpha B^n$, where $A = A(t)$ and $B = B(t)$ and α, n are real constants. This paper is organized as follows: The metric and field equations are considered in §2. Solutions of field equations are obtained in §3. §4 deals some important physical and geometrical features of the model. In last §5, conclusions are given.

2. Metric and field equations

We consider the LRS Bianchi type I metric in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)(dy^2 + dz^2), \quad \dots (1)$$

where the metric potentials A and B are functions of time (t) alone.

The energy momentum tensor has the form

$$T_i^j = (\rho + p)v_i v^j + p g_i^j, \quad \dots (2)$$

In equation (2), ρ is energy density, p is the pressure and v^i is the four velocity vector satisfying the relation

$$g_{ij}v^i v^j = -1. \quad \dots (3)$$

The energy momentum tensor T_i^j corresponding to zero mass scalar fields ϕ is given by

$$T_i^j = \phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}, \quad \dots (4)$$

where $\phi(t)$ (a function of t only) is the zero mass scalar fields which satisfies the wave equation

$$g^{ij}\phi_{;ij} = 0. \quad \dots (5)$$

The Einstein's field equations with time dependent G and Λ are

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi GT_i^j - \Lambda g_i^j, \quad \dots (6)$$

where the contracted tensor T is trace of energy momentum tensor describing all non-gravitational and non-scalar field matter and energy. We assume the coordinates to be commoving so that

$$v^1 = v^2 = v^3 = 0 \quad \text{and} \quad v^4 = 1, \quad \dots (7)$$

For the line element (1) the field equations (4) and (6) lead to the following system of equations

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} + \frac{\phi_4^2}{2} = -8\pi Gp + \Lambda, \quad \dots (8)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} + \frac{\phi_4^2}{2} = -8\pi Gp + \Lambda, \quad \dots (9)$$

$$\frac{2A_4B_4}{AB} + \frac{B_4^2}{B^2} - \frac{\phi_4^2}{2} = 8\pi G\rho - \Lambda, \quad \dots (10)$$

$$\phi_{44} + \phi_4 \left(\frac{A_4}{A} + \frac{2B_4}{B} \right) = 0, \quad \dots (11)$$

An additional equation for time changes of G and Λ are obtained by the divergence of Einstein's tensor i.e. $(R_i^j - \frac{1}{2}Rg_i^j)_{;j} = 0$ which leads to

$$(8\pi GT_i^j - \Lambda g_i^j)_{;j} = 0 \quad \dots (12)$$

which yields

$$8\pi G_4\rho + \Lambda_4 + 8\pi G \left[\rho_4 + (\rho + p) \left(\frac{A_4}{A} + \frac{2B_4}{B} \right) \right] = 0 \quad \dots (13)$$

From equation (12) we have

$$\rho_4 + (\rho + p) \left(\frac{A_4}{A} + \frac{2B_4}{B} \right) = 0, \quad \dots (14)$$

$$\text{and } 8\pi G_4\rho + \Lambda_4 = 0 \quad \dots (15)$$

where the sub-index 4 in A and B denotes ordinary differentiation with respect to t. The velocity field v^j is irrotational. The scalar expansion θ and components of shear σ_i^j are given by

$$\theta = \frac{A_4}{A} + \frac{2B_4}{B}, \quad \dots (16)$$

$$\sigma_1^1 = \frac{2}{3} \left[\frac{A_4}{A} - \frac{B_4}{B} \right], \quad \dots (17)$$

$$\sigma_2^2 = \frac{1}{3} \left[\frac{B_4}{B} - \frac{2A_4}{A} \right], \quad \dots (18)$$

$$\sigma_3^3 = \frac{1}{3} \left[\frac{B_4}{A} - \frac{A_4}{A} \right], \quad \dots (19)$$

$$\sigma_4^4 = 0. \quad \dots (20)$$

Therefore the shear (σ) is defined as

$$\sigma = \frac{1}{\sqrt{3}} \left[\frac{A_4}{A} - \frac{B_4}{B} \right]. \quad \dots (21)$$

3. Solutions of the field equations

Equations (8)-(12) are five independent equations in seven unknowns A, B, p, G, Λ , ρ and ϕ for complete determinacy of the system, we need two extra conditions:

(i) The equation of state $p=0$ responding dust filled model.

$$\sigma \propto \theta \text{ leads to } A = \alpha B^n, \quad \dots (22)$$

where α and n are real constants.

From equations (8) and (9) we obtain

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} - \frac{B_4^2}{B^2} + \frac{A_4 B_4}{AB} = 0. \quad \dots (23)$$

Equation (19) on differentiation gives

$$\frac{A_4}{A} = \frac{nB_4}{B}, \quad \dots (24)$$

$$\text{and } \frac{A_{44}}{A} = \frac{n(n-1)B_4^2}{B^2} + \frac{nB_{44}}{B}. \quad \dots (25)$$

Using equations (21) and (22) in equation (20), we have

$$\frac{B_{44}}{B} + \frac{(n+1)B_4^2}{B^2} = 0. \quad \dots (26)$$

Equation (23) on integration leads to

$$B^{n+1}B_4 = c_1, \quad \dots (27)$$

where c_1 is constant of integration,

which on again integration gives

$$B = \left[(n+2)(c_1t + c_2) \right]^{\frac{1}{n+2}}. \quad \dots (28)$$

where c_2 is constant of integration.

Using equations (25) in equation (19), we obtain

$$A = \alpha \left[(n+2)(c_1t + c_2) \right]^{\frac{n}{n+2}}. \quad \dots (29)$$

Using equations (25) and (26) in equation (11) we have

$$\frac{\phi_{44}}{\phi_4} = \frac{-c_1}{(c_1t + c_2)}, \quad \dots (30)$$

which on integration gives

$$\phi_4 = \frac{c_3}{(c_1t + c_2)}. \quad \dots (31)$$

where c_3 is integrating constant.

Equation (31) on integration leads to

$$\exp(\phi) = \phi_0 (c_1t + c_2)^{\frac{c_3}{c_1}}, \quad \dots (32)$$

where ϕ_0 is constant of integration.

Hence the metric (1) leads to

$$ds^2 = -dt^2 + \left[\alpha \{ (n+2)(c_1t + c_2) \}^{\frac{2n}{n+2}} \right] dx^2 + \{ (n+2)(c_1t + c_2) \}^{\frac{2}{n+2}} (dy^2 + dz^2) \quad \dots (33)$$

After suitable transformation of coordinates metric (29) reduces to the form

$$ds^2 = -\frac{dT^2}{c_1^2} + \left[\alpha \{ (n+2)T \}^{\frac{2n}{n+2}} \right] dX^2 + [(n+2)T]^{\frac{2}{n+2}} (dY^2 + dZ^2) \quad \dots (34)$$

Equation (30) represents the LRS Bianchi type I dust filled cosmological model with zero- mass scalar fields given by

$$\phi = \log \left[\phi_0 (T)^{\frac{c_3}{c_1}} \right], \quad \dots (35)$$

which is not defined at $T = 0$.

4. Some Physical and Geometrical Features

The energy density ρ for the dust filled model (31) is given by

$$\rho = \frac{k_1}{(n+2)T}, \quad \dots (36)$$

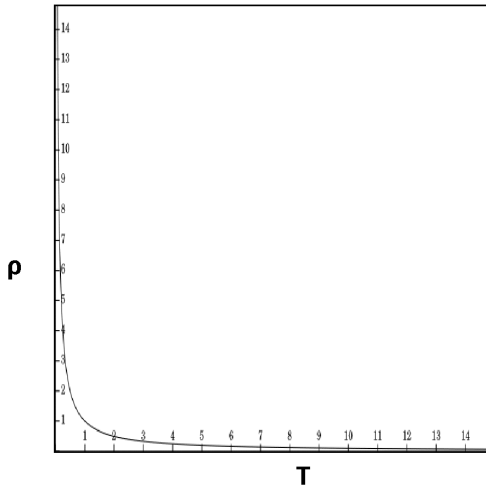


Fig. 1 Density with Time when $n = -1$

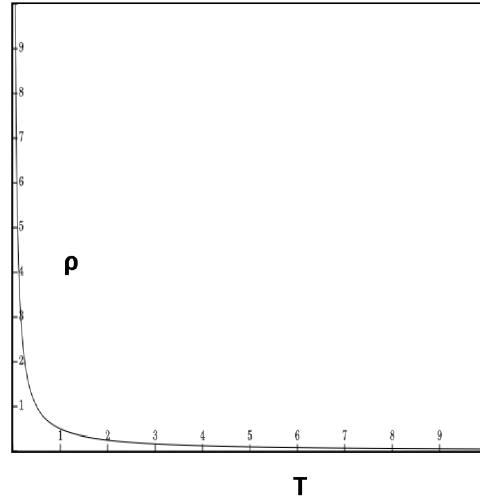


Fig. 2 Density with Time when $n = 0$

Figure 1 and 2 show the behavior of energy density with time for $n = -1$ and $n = 0$ respectively. From the figures we find that $\rho \rightarrow 0$ as $T \rightarrow \infty$ and $\rho \rightarrow \infty$ as $T \rightarrow 0$. This curve is asymptotic to ρ as well as T . The energy condition $\rho \geq 0$ leads to $k_1 \geq 0$ & $n > -2$.

The expansion (θ) and shear (σ) for the model (31) are given by

$$\theta = \frac{c_1}{T}, \tag{37}$$

$$\sigma = \frac{1}{\sqrt{3}} \left[\frac{c_1 \sqrt{(n^2 - 1)}}{(n + 2)T} \right], \tag{38}$$

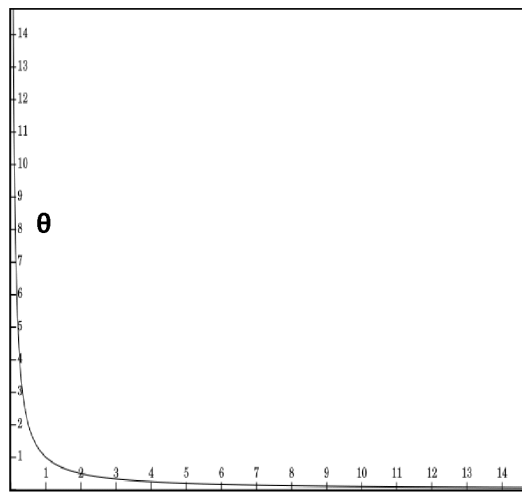


Fig. 3 Expansion with Time

Figure 3 shows the asymptotic behavior of the model (31), thus expansion is inversely proportional with time. Hence $\theta \rightarrow \infty$ as $T \rightarrow 0$ and $\theta \rightarrow 0$ as $T \rightarrow \infty$.

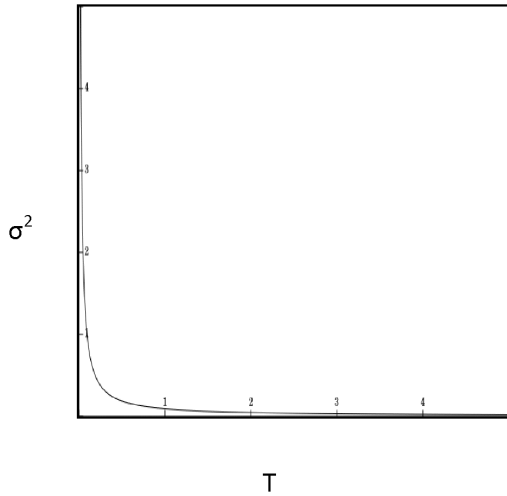


Fig. 4 Shear with Time when n=2

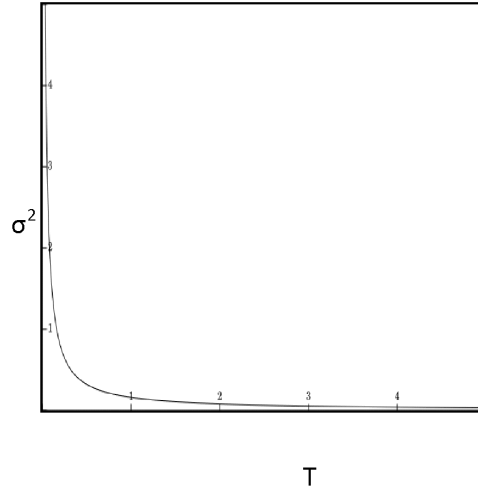


Fig. 5 Shear with Time when n=3

Figure 4 and 5 exhibits the nature of shear with time for n=2 and n=3 respectively. We can observe that $\sigma \rightarrow 0$ as $T \rightarrow \infty$ and $\sigma \rightarrow \infty$ as $T \rightarrow 0$.

The expressions $\frac{\sigma}{\theta}$ and $\frac{\rho}{\theta^2}$ for the model (31) is calculated as follows:

$$\frac{\sigma}{\theta} = \frac{(n^2 - 1)^{\frac{1}{2}}}{\sqrt{3}(n + 2)}, \tag{39}$$

and $\frac{\rho}{\theta^2} = \frac{k_1 T}{(n + 2)c_1^2}, \tag{40}$

since $\lim_{T \rightarrow \infty} \frac{\sigma}{\theta} \neq 0$, hence the model does not approach isotropy for large values of T.

Since $\frac{\rho}{\theta^2} \rightarrow \infty$ as $T \rightarrow \infty$ with $n > -2$ which shows that matter density dominates the expansion at late times.

The cosmological constant Λ is given by

$$\Lambda = \frac{1}{T^2} \left[\frac{c_3^2}{2} - \frac{nc_1^2}{(n+2)^2} \right]. \quad \dots (41)$$

From equation (15)

$$G_4 = \frac{c_4}{T^2} \quad \dots (42)$$

$$\text{Where } c_4 = 2c_1 \left(\frac{c_3^2}{2} - \frac{nc_1^2}{(n+2)^2} \right)$$

Equation on integration gives

$$G = -\frac{c_4}{T} + c_5 \quad \dots (43)$$

where c_5 is constant of integration and $c_4 < 0$.

In the model we observe that the cosmic scenario starts from a Big Bang at $T = 0$ and continues till $T = \infty$. The model has singularity at $T=0$ (Maccullum, [12]). Near the singularity $T = 0$ the physical parameters ρ, G, σ^2 and θ are all infinite and monotonically decreasing when T increases and becomes zero when $T \rightarrow \infty$. When $T \rightarrow \infty$ then $\Lambda \rightarrow 0$ i.e. cosmological term becomes negligible at later time. Therefore for realistic model we take $k \geq 0$ and $n > -2$. The gravitational constant is inversely proportional to time. We observe that Λ decays faster than G .

The density parameter is obtained as

$$\Omega = \frac{8\pi G\rho}{3H^2} = \frac{24\pi k_1 T}{c_1^2} \left(c_5 - \frac{c_4}{T} \right), \quad \dots (44)$$

$$\Omega_A = \frac{\Lambda}{3H^2} = \frac{3}{c_1^2} \left[\frac{c_3^2}{2} - \frac{nc_1^2}{(n+2)^2} \right]. \quad \dots (45)$$

The ratio between vacuum and matter densities scale as

$$\frac{\Lambda}{8\pi G\rho} = \frac{3\{c_3^2(n+2)^2 - 2nc_1^2\}}{8\pi\{(n+2)(Tc_5 - c_4)k_1\}} \dots (46)$$

The spatial volume (V) is evaluated as

$$V = R^3 = AB^2 = \alpha(n+2)T, \dots (47)$$

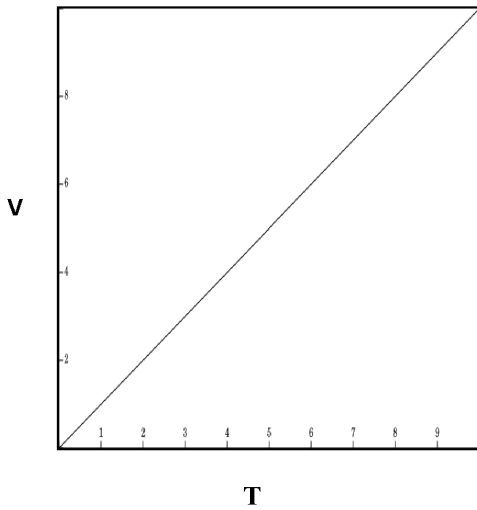


Fig. 6 Spatial Volume with Time (n=-1)

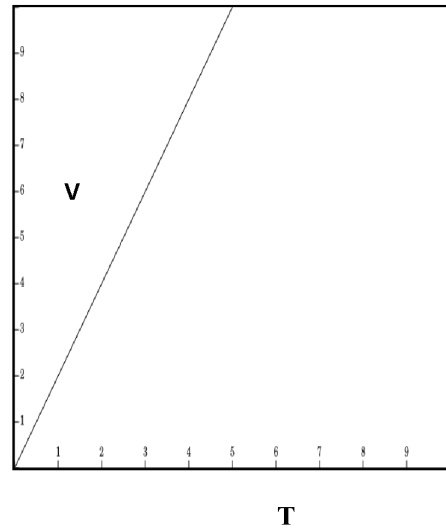


Fig. 7 Spatial Volume with Time (n=0)

The spatial volume V increases with time. When $T \rightarrow \infty$ then $V \rightarrow \infty$. From the figures 6 and 7 we find that V and T increases equally at $n = -1$ whereas V increases faster than T at $n = 0$. Thus inflation is possible in LRS Bianchi type I cosmological model with a mass less scalar fields.

Average scale factor R(t) for the model (31) is given by

$$R = \{a(n+2)T\}^{\frac{1}{3}} \dots (48)$$

The deceleration parameter (q) is given by

$$q = -\frac{R_{44}R}{R_4^2} = 2 \dots (49)$$

which is positive hence the model decelerates.

The components of Hubble parameter along x,y and z axes are given by

$$H_1 = \frac{A_4}{A} = \frac{nc_1}{(n+2)T}, \quad \dots (50)$$

$$H_2 = H_3 = \frac{B_4}{B} = \frac{c_1}{(n+2)T}, \quad \dots (51)$$

hence the Hubble parameter H is given by

$$H = \frac{c_1}{3T}. \quad \dots (52)$$

The anisotropy parameter \bar{A} is given by

$$\bar{A} = \frac{1}{3} \sum_{i=1}^3 \left(\frac{\Delta H_i}{H} \right)^2, \quad \dots (53)$$

where $\Delta H_i = H_i - H (i=1,2,3)$.

For our model the anisotropy parameter \bar{A} is given by

$$\bar{A} = \frac{1}{3} \left[\frac{(n^2 + 2)}{(n+2)^2} + \frac{1}{3} - \frac{2(n+1)}{3(n+2)} \right]. \quad \dots (54)$$

The expansion velocity R_4 is given by

$$R_4 = \frac{[\alpha(n+2)]^{\frac{1}{3}}}{3T^{\frac{2}{3}}}. \quad \dots (55)$$

For the model (30) the particle horizon exists because

$$\int_{t_0}^T \frac{dt}{R(t)} = \left[\frac{3(c_1 t + c_2)^2}{2c_1 [\alpha(n+2)]^{\frac{1}{3}}} \right]_{t_0}^T \quad \dots (56)$$

is finite i.e. the particles are in communicable region.

5. Conclusion :

In this paper we have presented the LRS Bianchi type I dust filled cosmological model in the presence of zero- mass scalar fields with varying G and Λ term. The model expands with a big-bang at $T = 0$ i.e. $t = -c_2/c_1 = t_0$ with $n > -1$ and stops at $T = \infty$ with $n > -1$. Volume V increases as time increases. Shear $\sigma = 0$ as $T \rightarrow \infty$ and $\sigma = \infty$ as $T \rightarrow 0$. Model exhibits initial singularity (Maccullum, [12]) at $T=0$. Equation (33) shows that energy density diverges at $T \rightarrow 0$ and vanishes at $T \rightarrow \infty$. The scalar field is singular at $T=0$. Hubble parameters vanish at $T \rightarrow \infty$ and tends to ∞ as $T \rightarrow 0$. In the model cosmological term Λ becomes negligible at late times. Therefore for realistic modal we take $k \geq 0$ and $n > -2$. The gravitational constant is inversely proportional to time. To avoid the singularities we must have $T > 0$.

It is remarkable to note that energy density of the universe is a positive quantity. It is believed that at the early stages of the evolution when the average scale factor R was close to zero, the energy density of the universe was infinitely large. The cosmological constant for this model is found to be small and positive which are supported by the results from recent supernova Ia observations recently obtained by the High-Z Supernova Team and Supernova Cosmological Project (Garnavich et al.[6]; Perlmutter et al.[16]; Riess et al.[23]; and Schmidt et al.[25]). When $c_3 = 0$, the model (31) reduces to general relativity case with constant zero-mass scalar field.

Acknowledgement

The authors are thankful to the learned referee for the valuable comments and suggestions.

References

- [1] Bali,R. and Upadhaya, R.D. (2003). L.R.S. Bianchi type I string dust magnetized cosmological models, *Astrophys. Space Sci.* **283**, 97-108
- [2] Banerjee, A., Sanyal, A.K. and Chakraborty (1990). String cosmology in Bianchi I space-time, *Pramana J. Phys.* **34**, 1-11
- [3] Brahmchary, R. L. (1960). A solution of the combined Gravitational and mesic field equations in general relativity, *Prog. Theor. Phys.*, **23**, 749–750
- [4] Dirac, P.A.M. (1937). The Cosmological Constants, *Nature* **130**, 323-334 *Nature* **139**, 323 (20 February 1937) | doi:10.1038/139323a0
- [5] Dolgov, A.D. (1993). Breaking of conformal invariance and electromagnetic field generation in the Universe, *Phys. Rev. D*, **48**, 2499
- [6] Garnavich, P. M. et al. (1998). Constraints on cosmological models from Hubble space telescope observations of high-z supernovae, *Astrophys.J.* **493**, L53, astro-ph / 9710123 (1998a); *Astrophys. J.* **509**,74, astro-ph/9806396 (1998b)
- [7] Gautreau, R. (1969). Coupled weyl gravitational and zero-rest-mass scalar fields, *Nouv. Cim.* **B62** ,360-370
- [8] Hajj-Boutros and Sfeila, J. (1989). Zero-curvature FRW models and Bianchi I space-time as solutions of the same equation, *Int. J. Theor. Phys.* **26**, 97-103
- [9] Jain, V.C, and Yadav , M. K. (2010). LRS Bianchi type II disordered radiation model with a varying Λ term in self-creation theory, *Astrophys. Space Sci.* **331**, 343
- [10] Jain,V.C. (2009).Magnetized Bianchi Type-I inflationary cosmological model in general relativity, *Indian Acad. of Math* **31**, 369
- [11] Kalligas, D., Wesson, P. and Everitt, C.W. F. (1992). Flat FRW models with variable G and Λ , *Gen. Relativ. Gravit.* **24**, 351-357 D.
- [12] MacCallum, M.A.H. (1971). A class of homogeneous cosmological models III: Asymptotic behavior, *Communications in Math. Phys.* **20**, 57-84
- [13] Mazumdar, A. (1994). Solutions of LRS Bianchi I space-time filled with a perfect fluid, *Gen. Relativ. Gravit.* **26**, 307-310

- [14] Patel, L. K., and Trivedi, V.M. (1975). Some axially symmetric zero mass meson solutions of Einstein's equations, *J. Austral Math. Soc.* **19**, 140-145
- [15] Peebles, P. J.E., AND Ratra, B. (2003). The cosmological constant and dark energy, *Rev. Mod. Phys.* **75**, astro-ph / 0207347
- [16] Perlmutter, S. et al. (1997). Measurements* of the cosmological parameters Ω and Λ from the first seven supernovae at $z \geq 0.35$, *Astrophys. J.* **483**, 565, astro-ph / 9608192; *Nature* **391**, 51, astro-ph / 9712212 (1998) *Astrophys. J.* **517**, 565, astro-ph / 9608192 (1999)
- [17] Pradhan, A. and Pandey, H. R. (2003) Plane-symmetric inhomogeneous bulk viscous cosmological models with variable Λ , *Int. J. Mod. Phys.* **D12**, 941-950
- [18] Pradhan, A., and Pandey, P. (2006). Some Bianchi type I viscous fluid cosmological models with a variable cosmological constant, *Astrophys. Space Sci* **301**, 127-134
- [19] Pradhan, A., Tiwari, K L., and Beesham, A. : (2001) LRS Bianchi I cosmological models in the presence of zero-mass scalar field. *Indian J. Pure Appl. Math.* **32(6)**, 789
- [20] Rao, V. U. M, Sanyasiraju, Y. V. S. S, Reddy, D. R. K. and Venkeshwarlu (1987). Exact Bianchi-type VIII and IX models in the presence of zero-mass scalar fields *Astrophys. Space Sci.* **136**, 17-24
- [21] Reddy, D.R.K. (1988). A plane-symmetric universe in the presence of zero-mass scalar fields, *Astrophys. Space Sci.* **140**, 161
- [22] Rehman, A.M. M. (1990). A critical density cosmological model with varying gravitational and cosmological "constants" A, *Gen. Relativ. Gravit.* **22**, 655
- [23] Riess, Adam G. et al. (1998). Observational evidence from supernovae for an accelerating universe and a cosmological constant, *Astron. J.* **116**, 1009-1030, astro-ph/9805201.
- [24] Sahni, V., and Starobinsky, A. (2000). The case for a positive cosmological Λ - term, *Int. J. Mod. Phys.* **D 09**, 373-381 DOI: 10.1142/S0218271800000542

- [25] Schmidt, Brian P. et al (1998). The high-Z supernova search: measuring cosmic deceleration and global curvature of the universe using type Ia supernovae*, *Astrophys. J.* **507**, 46, astro-ph / 9805200
- [26] Singh, G.P. and Kale, A.Y. (2009). Anisotropic bulk viscous cosmological models with variable G and Λ , *Int. J. Theor. Phys.* **48**, 1177
- [27] Singh, G. P. and Kotambkar, S. (2001). Higher dimensional cosmological model with gravitational and cosmological "constants" *Gen. Relativ. Gravit.* **33**, 621-630
- [28] Singh, N.I. and Sorokhaibam, A. (2007). Cosmological models with the variable gravitational and cosmological constants, *Astrophys. Space Sci.* **310**, 131
- [29] Sri Ram(1989). LRS Bianchi type I perfect fluid solutions generated from known solutions, *Int. J. Theor. Phys.* **28**, 917
- [30] Yadav, A.K., Pradhan A. and Singh, A.K. (2012). Bulk viscous LRS Bianchi-I Universe with variable G and decaying Λ , *Astrophys. Space Sci.* **337**, 379