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TRANSIENT ANALYSIS OF A SINGLE SERVER LOSS AND DELAY BULK SERVICE MARKOVIAN QUEUE WITH STATE DEPENDENT RATES UNDER N-POLICY

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Abstract : In the present paper, the transient analysis of loss and delay bulk service Markovian queue has been done by considering the state dependent rates. The customers arrive in the system according to Poisson fashion and arrival rate depends upon the server's state. The single server renders service under the N-policy according to which when there are N customers accumulated in the system, the service starts and ends when system becomes empty. The loss and delay phenomena of the customers occur in the system due to the category of the customers depending upon whether they would like to wait in the queue when all the servers are busy with some other customers. The discouraging behavior of customers is also incorporated, to make our model more realistic to the real world situations. The customers are served singly up-to a threshold level, beyond that all customers are taken in a group for service. The service times are exponentially distributed. We employ the matrix method to solve Chapman Kolmogorov equations governing the underlying birth death process in terms of appropriate state dependent transient rates. The expressions for the average queue length, expected idle time, throughput, etc. are established. We investigate the optimal value of threshold parameter N in order to minimize the expected total cost. The sensitivity analysis is also carried out to examine the effect of various parameters on the system performance.

Keywords: N-Policy, Loss and delay, Batch service, State dependent rates, Markovian queue, Balking, Reneging, Matrix method.

2010 Mathematics Subject Classification: 90B22, 60K25.

1. Introduction

Bulk service queueing system is of interest from the view point of practical applications in real life congestion problems and are often encountered in day to day as well as industrial scenario; to illustrate we refer the transportation processes involving aeroplanes, trains, buses, ships; manufacturing, and production systems, assembly lines, computer communication systems, etc. A state dependent bulk service queue with balking was studied by Jain and Dhyani [5]. Cai and Zhen [1] developed increasing convex ordering of queue length in bulk queue. Goswami and Samanta [3] proposed discrete-time bulk-service queue with two heterogeneous servers.

There are several queueing situations in which the server can not provide service to the arriving customers until a prespecified number (say N) of customers is not accumulated in the system. This type of queueing system is known as N-policy. There is a great deal of literature on queueing models concerning N-policy. The N-policy for an unreliable server with delaying repair and two phases of service was obtaine by Choudhury et al. [2]. Park et al. [12] analysed a two-phase queueing system with a fixed-size batch policy.

Loss and delay phenomena (which is also called cut off priority) of customers is considered by many researchers. In some queueing situations, the customers may depart from the system due to unavailability of space or due to some other reason are called the loss customers; on the other hand remaining customers who have patience to wait in the system to get service are called the delay customers. Jain [4] developed a finite population loss and delay queueing system with no passing. Kim et al. [10] considered erlang loss queueing system with batch arrivals operating in a random environment. Network queue and loss analysis using histogram-based traffic models was analysed by Orallo and Carbo [11].

In real life, there are many queueing situations in which there may be tendency of customers to be discouragement by a long queue. Due to discouragement the arriving customers may balk or renege as long waiting times are undesirable. Jain and Sharma [6] obtained queueing characteristics for finite controllable Markovian queue with balking and reneging. Controllable multi server queue with balking was taken into consideration by Jain and Sharma [7] Finite capacity queueing system with queue dependent servers and discouragement was analysed by Jain and Sharma [8]. Jeffery and Jams [9] developed nonlinear queuing regressions to increase emergency department patient safety: Approximating reneging with balking.

In this paper, we focus our attention on the queueing model with single and bulk service by incorporating loss and delay phenomena under N-policy. However, as usually happens in real practical problems, the arriving customers may balk and renege from the system. The server starts service only when N customers are accumulated in the system and stops when the system becomes empty. The remaining paper is organized as follows. §2 provides the mathematical formulation by stating underline assumptions of the model. The transient analysis of the model is given in §3. The performance characteristics of the system are obtained in §4. The cost analysis and sensitivity analysis are given in sections §5 and §6, respectively. In the final §7, conclusions are drawn and future scope of the work is also highlighted.

2. The model description

Consider a finite queueing model with loss and delay, single and bulk service under N-policy. The customers arrive at the system in accordance with a Poisson process. After a threshold level C, the arriving customers may join (balk) with probability β ($\overline{\beta} = 1 - \beta$). However after a threshold level L when loss customers are not allowed to join the queue, the server facilitates service in group mode, and the delay customers may also join (balk) with probability β' ($\overline{\beta'} = 1 - \beta'$). On finding the server busy, the customers may renege according to exponential distribution with parameter α. The service discipline is first come first serve (FCFS). The capacity of the system is K.

The state dependent arrival rate is defined as follows:

$$\lambda_n = \begin{cases} \lambda; & 0 \le n < C \\ \lambda\beta; & C \le n < L \\ \lambda_d \beta'; & L \le n \le K \end{cases} \qquad \text{.... (1)}$$

Here $\lambda = \lambda_\ell + \lambda_d$, where λ_ℓ is the arrival rate of loss customers and λ_d is the arrival rate of delay customers.

At any time t, the status of the server is represent by $\xi(t)$ as follows:

$$\xi(t) = \begin{cases} 0; & \text{when server is idle.} \\ 1; & \text{when server is rendering service in single mode with normal rate } \mu. \\ 2; & \text{when server is rendering service in single mode with faster rate } \mu_f. \\ 3; & \text{when server is rendering service in batch mode with service rate } \mu_b. \end{cases}$$

The service rate is given by

$$\mu_n = \begin{cases} \mu + (n-1)\alpha; & 1 \le n \le C \\ \mu_f + (n-1)\alpha; & C < n \le L \\ \mu_b + (n-1)\alpha; & L < n \le K \end{cases} \qquad \text{....}(2)$$

Here μ_f is the faster rate of service in case when the number of customers reaches to a threshold level C, and μ_b is the service rate when the server switches to bulk service mode after second threshold level L. in batch mode, the server renders service to all customers present in the system i.e. the system works as infinite server system. The probabilities in different states are defined as follows:

$P_{i,n}(t)$= Pr{n customers in the system at time t and $\xi(t)$=i .

3. The Transient Analysis

The transient state equations governing the model are constructed as follows:

$$\frac{dP_{0,0}(t)}{dt} = -\lambda P_{0,0}(t) + \mu P_{1,1}(t) + \sum_{i=L+1}^{K} \mu_i P_{3,i}(t) \qquad \text{....}(3)$$

$$\frac{dP_{0,n}(t)}{dt} = -\lambda P_{0,n}(t) + \lambda P_{0,n-1}(t);\ 1 \le n \le N-1 \qquad \text{....}(4)$$

$$\frac{dP_{1,1}(t)}{dt} = -(\lambda + \mu)P_{1,1}(t) + (\mu + \alpha)P_{1,2}(t) \qquad \text{....}(5)$$

$$\frac{dP_{1,n}(t)}{dt} = -(\lambda + \mu + (n-1)\alpha)P_{1,n}(t) + \lambda P_{1,n-1}(t) + (\mu + n\alpha)P_{1,n+1}(t);\ 2 \le n \le N-1 \qquad \text{....}(6)$$

$$\frac{dP_{1,N}(t)}{dt} = -(\lambda + \mu + (N-1)\alpha)P_{1,N}(t) + \lambda P_{0,N-1}(t) + \lambda P_{1,N-1}(t) + (\mu + N\alpha)P_{1,N+1}(t) \qquad \text{....}(7)$$

$$\frac{dP_{1,n}(t)}{dt} = -(\lambda + \mu + (n-1)\alpha)P_{1,n}(t) + \lambda P_{1,n-1}(t) + (\mu + n\alpha)P_{1,n+1}(t);\ N+1 \le n < C \qquad \text{....}(8)$$

$$\frac{dP_{1,C}(t)}{dt} = -(\lambda\beta + \mu + (C-1)\alpha)P_{1,C}(t) + \lambda P_{1,C-1}(t) + (\mu_f + C\alpha)P_{2,C+1}(t) \qquad \text{....}(9)$$

$$\frac{dP_{2,n}(t)}{dt} = -(\lambda\beta + \mu_f + (n-1)\alpha)P_{2,n}(t) + \lambda\beta P_{2,n-1}(t) + (\mu_f + n\alpha)P_{2,n+1}(t);$$

$$C+1 \le n \le L-1 \qquad \text{.... (10)}$$

$$\frac{dP_{2,L}(t)}{dt} = -(\lambda_d\beta' + \mu_f + (L-1)\alpha)P_{2,L}(t) + \lambda_d\beta' P_{2,L-1}(t) + (\mu_b + L\alpha)P_{3,L+1}(t) \qquad \text{.... (11)}$$

$$\frac{dP_{3,n}(t)}{dt} = -(\lambda_d\beta' + \mu_b + (n-1)\alpha)P_{3,n}(t) + \lambda_d\beta' P_{3,n-1}(t) + (\mu_b + n\alpha)P_{3,n+1}(t);$$

$$L+1 \le n \le K-1 \qquad \text{.... (12)}$$

$$\frac{dP_{3,K}(t)}{dt} = -(\mu_b + (K-1)\alpha)P_{3,K}(t) + \lambda_d\beta' P_{3,K-1}(t) \qquad \text{.... (13)}$$

Taking Laplace transform of the above equations (3)-(13) and using the initial condition $P_{0,0}(0)=1$; $P_{i,j}(0)=0$, $i \neq 0$, $j \neq 0$, we have

$$(s+\lambda)\tilde{P}_{0,0}(s) - 1 = \mu\tilde{P}_{1,1}(s) + \sum_{i=L+1}^{K} \mu_i \tilde{P}_{3,i}(s) \qquad \text{.... (14)}$$

$$(s+\lambda)\tilde{P}_{0,n}(s) = \lambda\tilde{P}_{0,n-1}(s);\ 1 \le n \le N-1 \qquad \text{.... (15)}$$

$$(s+\lambda+\mu)\tilde{P}_{1,1}(s) = (\mu+\alpha)\tilde{P}_{1,2}(s) \qquad \text{.... (16)}$$

$$(s+\lambda+\mu+(n-1)\alpha)\tilde{P}_{1,n}(s) = \lambda\tilde{P}_{1,n-1}(s) + (\mu+n\alpha)\tilde{P}_{1,n+1}(s);\ 2 \le n \le N-1 \qquad \text{.... (17)}$$

$$(s+\lambda+\mu+(N-1)\alpha)\tilde{P}_{1,N}(s) = \lambda\tilde{P}_{0,N-1}(s) + \lambda\tilde{P}_{1,N-1}(s) + (\mu+N\alpha)\tilde{P}_{1,N+1}(s) \qquad \text{.... (18)}$$

$$(s+\lambda+\mu+(n-1)\alpha)\tilde{P}_{1,n}(s) = \lambda\tilde{P}_{1,n-1}(s) + (\mu+n\alpha)\tilde{P}_{1,n+1}(s);\ N+1 \le n < c \qquad \text{.... (19)}$$

$$(s+\lambda\beta+\mu+(C-1)\alpha)\tilde{P}_{1,c}(s) = \lambda\tilde{P}_{1,c-1}(s) + (\mu_f + C\alpha)\tilde{P}_{2,c+1}(s) \qquad \text{.... (20)}$$

$$(s+\lambda\beta+\mu_f+(n-1)\alpha)\tilde{P}_{2,n}(s) = \lambda\beta\tilde{P}_{2,n-1}(s) + (\mu_f + n\alpha)\tilde{P}_{2,n+1}(s);\ C+1 \le n \le L-1 \qquad \text{.... (21)}$$

$$(s+\lambda_d\beta'+\mu_f+(L-1)\alpha)\tilde{P}_{2,L}(s) = \lambda_d\beta'\tilde{P}_{2,L-1}(s) + (\mu_b + L\alpha)\tilde{P}_{3,L+1}(s) \qquad \text{.... (22)}$$

$$(s+\lambda_d\beta'+\mu_b+(n-1)\alpha)\tilde{P}_{3,n}(s) = \lambda_d\beta'\tilde{P}_{3,n-1}(s) + (\mu_b + n\alpha)\tilde{P}_{3,n+1}(s);$$

$$L+1 \le n \le K-1 \qquad \text{.... (23)}$$

$$(s+\mu_b+(K-1)\alpha)\tilde{P}_{3,K}(s) = \lambda_d\beta'\tilde{P}_{3,K-1}(s) \qquad \text{.... (24)}$$

Equations (14)-(24) can be written in the matrix form as

$$A(s)\tilde{P}(s) = P(0) \qquad \text{.... (25)}$$

$$\widetilde{P}(s)=\left[\widetilde{P}_0(s),\widetilde{P}_1(s),\widetilde{P}_2(s),\widetilde{P}_3(s)\right]^T$$

where

$$\widetilde{P}_0(s)=\left[\widetilde{P}_{0,0}(s),\widetilde{P}_{0,1}(s),\widetilde{P}_{0,2}(s),\ldots.,\widetilde{P}_{0,N-1}(s)\right]$$

$$\widetilde{P}_1(s)=\left[\widetilde{P}_{1,1}(s),\widetilde{P}_{1,2}(s)\ldots,\widetilde{P}_{1,C}(s)\right]$$

$$\widetilde{P}_2(s)=\left[\widetilde{P}_{2,C+1}(s),\widetilde{P}_{2,C+2}(s)\ldots,\widetilde{P}_{2,L}(s)\right]$$

$$\widetilde{P}_3(s)=\left[\widetilde{P}_{3,L+1}(s),\widetilde{P}_{3,L+2}(s)\ldots,\widetilde{P}_{3,K}(s)\right]$$

$$P(0)=\left[\widetilde{P}_0(0),\widetilde{P}_1(0),\widetilde{P}_2(0),\widetilde{P}_3(0)\right]^T$$

Here A(s) is the coefficient matrix of order $(N+K)\times(N+K)$.

Define

$$j=\begin{cases} n; & 0\le n<N,\ i=0 \\ N+n; & N\le n<N+K,\ i=1,2,3 \end{cases}$$

Using Cramer's rule, $\widetilde{P}(s)$ can be determined as follows.

$$\widetilde{P}_{i,j}=\frac{|A_j(s)|}{|A(s)|};\ i=0,1,2,3 \ and\ j=0,1,2,\ldots.,N+K-1. \qquad \ldots.(26)$$

where $|A(s)|$ is a determinant of matrix A(s) and $\left|A_j(s)\right|$ is the determinant obtained by replacing jth (j = 0,1, 2,….,N+K-1) column of matrix A(s) with initial vector $P_0(0) = [1,0,0,\ldots.,0]^T$.

We know that s = 0 is a root of $|A(s)|=0$. Now we assume that $s=-\theta$, so that we get

$$A(-\theta)=A-\theta I, \qquad \ldots.(27)$$

where I is the identity matrix and A=A(0) in $(N+K)\times(N+K)$ matrix. Substituting the value of $A(-\theta)$ in (25), we get $A(-\theta)\widetilde{P}(s)=(A-\theta I)\widetilde{P}(s)=P(0)$ …. (28)

The distinct eigen values $\theta_j\ (\theta_j\neq 0\, where\ j=1,2,\ldots.,N+K-1)$ of the matrix A-θI, can be obtained by equating its determinant to zero. The eigen values can be real or complex.

Now let us suppose that there are p real eigen values (excluding zero) denoted by $\theta_1, \theta_2, \theta_3, .., \theta_p$. and q pair of distinct conjugate complex eigen values denoted by $(\theta_{p+1}, \bar{\theta}_{p+1}), (\theta_{p+2}, \bar{\theta}_{p+2}), ..., (\theta_{p+q}, \bar{\theta}_{p+q})$ where p+2q=N+K-1. Now $|A(s)|$ can be written as

$$|A(s)| = s\left[\prod_{k=1}^{p}(s+\theta_k)\right]\left[\prod_{k=1}^{q}\left\{s^2 + (\theta_{p+k} + \bar{\theta}_{p+k})s + \theta_{p+k}\bar{\theta}_{p+k}\right\}\right] \qquad \text{.... (29)}$$

and

$$\tilde{P}_{i,j}(s) = \frac{|A_j(s)|}{s\left[\prod_{k=1}^{p}(s+\theta_k)\right]\left[\prod_{k=1}^{q}\left\{s^2 + (\theta_{p+k} + \bar{\theta}_{p+k})s + \theta_{p+k}\bar{\theta}_{p+k}\right\}\right]} \qquad \text{.... (30)}$$

This implies

$$\tilde{P}_{i,j}(s) = \frac{a_0}{s} + \sum_{l=1}^{p}\frac{a_\ell}{s+\theta_l} + \sum_{l=1}^{q}\frac{b_l s + c_l}{s^2 + (\theta_{p+l} + \bar{\theta}_{p+l})s + \theta_{p+l}\bar{\theta}_{p+l}} \qquad \text{.... (31)}$$

where

$$a_0 = \frac{A_j(0)}{\left[\prod_{k=1}^{p}\theta_k\right]\left[\prod_{k=1}^{q}\theta_{p+k}\bar{\theta}_{p+k}\right]}$$

$$a_l = \frac{|A_j(-\theta_l)|}{(-\theta_l)\left[\prod_{\substack{k=1\\k\neq l}}^{p}(\theta_k - \theta_l)\right]\left[\prod_{\substack{k=1\\k\neq l}}^{q}\left\{\theta_l^{\,2} + (\theta_{p+l} + \bar{\theta}_{p+k})(-\theta_l) + \theta_{p+k}\bar{\theta}_{p+k}\right\}\right]}; \; l = 1,2,....,p.$$

and

$$b_l(-\theta_{p+l}) + c_l = \frac{|A_j(-\theta_{p+l})|}{(-\theta_{p+l})\left[\prod_{\substack{k=1\\k\neq l}}^{p}(\theta_k - \theta_{p+l})\right]\left[\prod_{\substack{k=1\\k\neq l}}^{q}\left\{(-\theta_{p+l})^2 + (\theta_{p+k} + \bar{\theta}_{p+k})(-\theta_{p+l}) + \theta_{p+k}\bar{\theta}_{p+k}\right\}\right]};$$

$$l = 1,2,....,q.$$

where a_0 and $a_l \; (l = 1,2,...., p); \; b_l \; and \; c_l \; (l = 1,2,...., q)$ are all real numbers.

The inverse Laplace transformation of equation (31) is given by

$$P_{i,j}(t)=a_0+\sum_{l=1}^{p}a_l e^{-\theta_l t}+\sum_{l=1}^{q}b_l e^{-u_l t}\cos(v_l t)+\frac{c_l+b_l u_l}{v_l}e^{-u_l t}\sin(v_l t) \quad \text{.... (32)}$$

where u_l *and* v_l denote the real and imaginary parts of the complex eigen value θ_{p+l}, *and* a_0, a_l, b_l *and* c_l are all real numbers.

4. System characteristics

The expected number of customers in the system at any instant t is expressed as

$$E\{N(t)\}=\sum_{n=1}^{N-1}nP_{0,n}(t)+\sum_{n=1}^{C}nP_{1,n}(t)+\sum_{n=C+1}^{L}nP_{2,n}(t)+\sum_{n=L+1}^{K}nP_{3,n}(t) \quad \text{.... (33)}$$

The probability that the server is being idle at time t, is

$$P\{I(t)\}=\sum_{n=0}^{N-1}P_{0,n}(t)=N\,P_{0,0}(t) \quad \text{.... (34)}$$

The throughput of the system $E(\tau_t)$is given by

$$E\{\tau(t)\}=\sum_{n=1}^{C}\mu_n P_{1,n}(t)+\sum_{n=C+1}^{L}\mu_n P_{2,n}(t)+\sum_{n=L+1}^{K}\mu_n P_{3,n}(t) \quad \text{.... (35)}$$

The probability that the server is being busy

$$P_B(t)=P_{B_1}(t)+P_{B_2}(t)+P_{B_3}(t) \quad \text{.... (36)}$$

where

$$P_{B_1}(t)=\sum_{n=1}^{C}P_{1,n}(t),\ P_{B_2}(t)=\sum_{n=C+1}^{L}P_{2,n}(t),\ \text{and}\ P_{B_3}(t)=\sum_{n=L+1}^{K}P_{3,n}(t)$$

The length of idle period is the sum of N exponential random variable having mean rate λ, so that

$$E\{I(t)\}=N/\lambda \quad \text{.... (37)}$$

The expected busy period of the system is obtained using

$$\frac{E\{B(t)\}}{E\{I(t)\}}=\frac{1-P\{I(t)\}}{P\{I(t)\}} \quad \text{.... (38)}$$

Then

$$E\{B(t)\}=\left(\frac{1-NP_{0,0}(t)}{NP_{0,0}(t)}\right)\left(\frac{N}{\lambda}\right)=\left(\frac{1-NP_{0,0}(t)}{\lambda P_{0,0}(t)}\right) \quad \text{.... (39)}$$

5. Cost Analysis

The following cost components to construct the cost function for proposed model are taken into consideration:

C_0	Set up cost.
C_h	Holding cost of per customer per unit time in the system.
C_1	Cost incurred when server is rendering normal service in single service mode.
C_2	Cost incurred per customer per unit time when server is rendering faster service in single service mode.
C_3	Cost incurred per customer per unit time when server is rendering bulk service.
C_4	Cost per unit time for turning the server on.
C_5	Cost per unit time for turning the server off.

The expected cost per unit time is given by

$$E\{C(t)\}=\frac{C_0}{E\{T(t)\}}+C_hE\{N(t)\}+C_1P_{B_1}(t)+C_2P_{B_2}(t)+C_3P_{B_3}(t)+(C_4+C_5)\frac{1}{E\{B(t)\}} \quad \text{.... (40)}$$

where $E\{T(t)\}$ is the expected cycle period and is defined as the sum of expected idle period and expected busy period.

Hence E{T(t)}=E{I(t)}+E{B(t)} .... (41)

The optimal value (say N^*) of the decision variable N, could be determined by setting

$$\frac{d\,E\{C(t)\}}{dN}=0 \quad \text{.... (42)}$$

In case when N^* is not an integer, then the best positive integer value N^* is achieved by rounding off the N^*.

To evaluate the analytic results for the optimum values of N, which minimize the expected total cost function, classical methods will not work as cost function is highly non-linear. For this purpose, heuristic search method based on discrete allocation scheme can be employed.

6. Sensitivity analysis

In this section, we provide the numerical results for various performance measures of the developed model by using MATLAB software. These results are summarized for different values of t and system capacity (K) in tables 1-6 by varying λ_d, λ_l, β, $\beta^{/}$, μ_f, and L respectively, for default parameter values of N=4, C=6, L=8, K=10, α=0.8, λ_d=1.8, λ_l=1.5, β=0.2, $\beta^{/}$=0.8, μ=1.5, μ_f=1.8, μ_b=1.9, C_0=20, C_h=30, C_1=35, C_2=40, C_3=45, C_4=100 and C_5=10.

We notice from tables 1-6 that expected number of customers in the system E{N(t)}, throughput of the system E{τ(t)} and probability that the server is busy P_B(t) increases but expected cost per unit time E{C(t)}decreases with the increase in the values of K and t, for fixed values of other parameters. This trend can be observed in many real time systems. From tables 1-4, it is clear that E{N(t)}, E{τ(t)}and P_B(t) increase while E{C(t)} decreases by increasing the values of λ_d, λ_l, β and $\beta^{/}$ for fixed values of K and t. As expected from table 5 a decreasing trend is observed for E{N(t)}, E{τ(t)} and P_B(t) whereas increasing trend is seen for E{C(t)} by varying μ_f. In table 6, a increasing trend is seen for E{N(t)} and E{C(t)} where as decreasing trends is observed for E{τ(t)} and P_B(t).

Finally, we conclude that, as per our expectation the expected number of customers in the system, the throughput and the probability of server being busy increases by increasing the arrival rates. The expected cost per unit time seems to decrease on increasing the arrival rates, joining probabilities, system capacity and time whereas increase in the service rate and threshold value results is increment in the expected cost per unit time.

7. Discussion

We have investigated the transient analysis of a single / bulk service Markovian queue with state dependent rate under N-policy. The discouraging behavior of the customers has been considered in our model which makes our model more closer to real life congestion situations. We have provided the computational tractable formulae for the system characteristic and done cost analysis which may be useful to practitioners in various fields such as manufacturing/production processes. The present model has potential utility in many real time systems such as communication networks, transportation, computer systems, etc.

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t	K	λ_l	E{N(t)}	E{τ(t)}	$P_B(t)$	$E\{C(t)\}$
0.5	15	0.1	1.39	0.29	0.219	549.46
		0.9	2.15	0.78	0.412	295.25
		1.9	3.18	2.01	0.652	199.50
	20	0.1	1.56	0.39	0.254	459.17
		0.9	2.45	1.11	0.408	231.51
		1.9	3.69	2.25	0.879	164.99
	25	0.1	1.65	0.39	0.191	388.84
		0.9	2.42	1.25	0.494	211.97
		1.9	3.55	2.38	0.925	135.77
1.5	15	0.1	2.19	1.29	0.579	142.83
		0.9	2.86	2.12	0.868	139.40
		1.9	3.64	2.79	1.99	134.92
	20	0.1	2.35	1.36	0.589	143.53
		0.9	3.01	2.21	0.868	134.40
		1.9	3.56	2.89	0.999	130.52
	25	0.1	2.25	1.39	0.697	144.78
		0.9	3.21	2.25	0.773	136.96
		1.9	3.59	2.78	0.998	125.97

Table 1 : Performance characteristics vs t by varying system capacity (K) and arrival rate of loss customers (λ_l)

t	K	λ_d	E{N(t)}	E{τ(t)}	$P_B(t)$	$E\{C(t)\}$
0.5	15	0.1	0.75	0.09	0.044	1680.87
		0.9	1.55	0.40	0.156	487.97
		1.9	3.33	1.69	0.692	189.73
	20	0.1	0.79	0.09	0.047	1478.94
		0.9	1.74	0.51	0.217	396.34
		1.9	3.65	2.22	0.898	165.44
	25	0.1	0.82	0.10	0.049	1389.29
		0.9	1.91	0.59	0.244	350.75
		1.9	3.25	2.15	0.891	155.67
1.5	15	0.1	1.69	0.71	0.357	199.24
		0.9	2.52	1.69	0.707	145.71
		1.9	3.86	2.75	1.055	146.50
	20	0.1	1.69	0.76	0.355	190.73
		0.9	2.56	1.71	0.797	145.83
		1.9	3.49	2.79	1.052	149.99
	25	0.1	1.71	0.78	0.350	189.64
		0.9	2.56	1.81	0.738	146.85
		1.9	3.52	2.92	1.078	149.38

Table 2: Performance characteristics vs t by varying system capacity (K) and arrival rate of delay customers (λ_d)

t	K	β	E{N(t)}	E{τ(t)}	$P_B(t)$	$E\{C(t)\}$
0.5	15	0.1	2.25	0.89	0.383	279.40
		0.3	2.69	1.15	0.444	235.61
		0.5	3.46	1.49	0.625	235.16
	20	0.1	2.49	1.12	0.441	229.08
		0.3	3.30	1.66	0.695	215.38
		0.5	5.76	3.35	1.294	202.35
	25	0.1	2.49	1.27	0.489	211.22
		0.3	3.15	1.72	0.702	172.05
		0.5	4.78	2.99	1.203	170.42
1.5	15	0.1	2.98	2.26	0.889	143.48
		0.3	3.15	2.45	0.969	142.81
		0.5	3.26	2.65	0.996	141.45
	20	0.1	3.10	2.31	0.910	142.89
		0.3	3.19	2.49	0.957	142.50
		0.5	3.32	2.65	0.998	142.42
	25	0.1	3.12	2.30	0.912	141.65
		0.3	3.25	2.45	0.992	141.51
		0.5	3.34	2.75	1.001	141.30

Table 3: Performance characteristics vs t by varying system capacity (K) and balking probability (β)

t	K	β'	E{N(t)}	E{τ(t)}	$P_B(t)$	$E\{C(t)\}$
0.5	15	0.1	1.59	0.46	0.208	482.64
		0.5	2.15	0.75	0.3.12	332.72
		0.9	2.69	1.25	0.456	235.06
	20	0.1	1.69	0.57	0.235	421.16
		0.5	2.35	0.95	0.388	269.29
		0.9	2.99	1.49	0.560	195.94
	25	0.1	1.85	0.66	0.256	384.29
		0.5	2.39	1.25	0.432	239.91
		0.9	2.76	1.59	0.656	180.40
1.5	15	0.1	2.75	1.99	0.791	149.47
		0.5	2.94	2.15	0.862	143.13
		0.9	3.19	2.37	0.939	141.59
	20	0.1	2.82	2.01	0.789	149.41
		0.5	2.95	2.19	0.876	144.32
		0.9	3.25	2.45	0.949	142.25
	25	0.1	2.85	2.09	0.804	148.86
		0.5	3.01	2.21	0.877	144.97
		0.9	3.25	2.41	0.949	146.20

Table 4 : Performance characteristics vs t by varying system capacity (K) and balking probability (β')

t	K	μ_f	E{N(t)}	$E\{\tau(t)\}$	$P_B(t)$	$E\{C(t)\}$
0.5	15	1.8	2.26	0.82	0.385	292.10
		2.8	1.81	0.59	0.224	441.53
		3.6	1.59	0.44	0.190	534.40
	20	1.8	2.60	1.15	0.421	236.91
		2.8	1.99	0.67	0.258	394.33
		3.6	1.58	0.42	0.133	528.79
	25	1.8	2.68	1.29	0.504	217.03
		2.8	2.12	0.72	0.302	336.92
		3.6	1.59	0.44	0.145	554.64
1.5	15	1.8	3.25	2.30	0.995	192.43
		2.8	2.99	2.03	0.814	143.82
		3.6	2.81	2.00	0.800	135.11
	20	1.8	3.55	2.39	0.956	151.89
		2.8	3.03	2.15	0.882	143.32
		3.6	2.25	2.03	0.823	134.91
	25	1.8	3.85	2.39	0.991	141.70
		2.8	3.01	2.25	0.837	149.07
		3.6	2.25	2.08	0.814	164.81

Table 5: Performance characteristics vs t by varying system capacity (K) and faster rate of service (μ_f)

t	K	L	E{N(t)}	$E\{\tau(t)\}$	$P_B(t)$	$E\{C(t)\}$
0.5	15	7	2.42	0.99	0.415	352.78
		9	3.88	0.69	0.279	450.95
		11	4.60	0.56	0.227	549.04
	20	7	2.77	1.29	0.593	203.76
		9	3.16	0.80	0.367	370.81
		11	4.85	0.60	0.204	443.03
	25	7	2.79	1.46	0.575	189.15
		9	3.24	1.32	0.366	240.78
		11	4.96	0.82	0.299	397.01
1.5	15	7	3.06	2.31	0.964	142.07
		9	4.89	2.25	0.805	155.16
		11	5.81	1.98	0.794	198.00
	20	7	3.08	2.39	0.965	141.61
		9	4.92	2.01	0.814	163.98
		11	5.85	1.12	0.707	186.21
	25	7	3.09	2.39	0.999	141.48
		9	4.94	2.11	0.852	173.48
		11	5.87	1.05	0.769	195.40

Table 6 : Performance characteristics vs t by varying system capacity (K) and threshold value (L)