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SELF-SIMILAR FLOW OF A DUSTY GAS BEHIND AN EXPONENTIAL SHOCK WITH HEAT CONDUCTION AND RADIATION HEAT FLUX

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Abstract : A self-similar solution for the propagation of a shock wave in a dusty gas with heat conduction and radiation heat flux is investigated. The shock is assumed to be driven out by a piston moving with time according to an exponential law and the dusty gas is assumed to be a mixture of non-ideal gas and small solid particles. The density of the ambient medium is assumed to be constant. The heat conduction is expressed in terms of Fourier's law and radiation is considered to be of diffusion type for an optically thick grey gas model. The thermal conductivity k and the absorption coefficient α_R are assumed to vary with temperature and density. Similarity solutions are obtained, and the effects of the variation of the parameter of non-idealness of the gas in the mixture, the mass concentration of solid particles and the ratio of density of solid particles to the initial density of the gas are investigated.

Keywords : Piston problem, self-similarity, exponential shock, dusty gas, heat conduction and radiation heat flux.

2010 Mathematics Subject Classification : 35C06, 76L05

1. Introduction

The study of shock wave in a mixture of a gas and small solid particles is of great importance due to its applications to nozzle flow, lunar ash flow, bomb blasts, coal-mine blasts, underground, volcanic and cosmic explosions, supersonic flight in polluted air, collision of a coma with a planet and many other engineering problems (see Pai et al [16]; Higashino and Suzuki [8]; Miura and Glass [12]; Gretler and Regenfelder [5]; Popel and Gisko [18]; Vishwakarma and Nath [30]; Vishwakarma et al [32]). Shock waves often arise in nature because of a balance between wave breaking non-linear and wave damping dissipative forces (Zel'dovich and Raizer [35]). Collisional and collisionless shock waves can appear because of friction between the particles and wave-particle interaction (Sagdeev [23]; Chen [2]) respectively. Miura and Glass [13] obtained an analytic solution for planar dusty gas flow with constant velocities of the shock and the piston moving behind it. As they neglected the volume occupied by the solid particles mixed into the perfect gas, the dust virtually has a mass fraction but no volume fraction. Their results reflect the influence of the additional inertia of dust on shock propagation. Pai et al [16] generalized the well known solution of a strong explosion due to an instantaneous release of energy in gas (Sedov [24]; Korobeinikov [9]) to the case of two-phase flow of a mixture of perfect gas and small solid particles, and brought out the essential effects due to the presence of dusty particles on such a strong shock wave. As they considered the non-zero volume fraction of solid particles in the mixture, their results reflect the influence of both the decrease of mixture compressibility and the increase of the mixture's inertia on shock propagation (Steiner and Hirschler [26]; Vishwakarma and Pandey [33]).

Marshak [11] studied the effects of radiation on the shock propagation by introducing the radiation diffusion approximation. Using the same mode of radiation, Elliott [3] discussed the conditions leading to self-similarity with a specified functional form of the mean free path of radiation and obtained a solution for self-similar spherical explosions. Wang [34], Helliwell [7] and Nicastro [14] treated the problem of radiating walls, either stationary or moving, generating shocks at the head of self-similar flow fields. Gretler and Wehle [6] studied the propagation of blast waves with exponential heat release by taking internal heat conduction and thermal radiation in a detonating medium. Ghoniem et al [4] obtained a self-similar solution for spherical explosions taking into account the effects of both conduction and radiation in the two limits of Rosseland radiative diffusion and Plank radiative emission.

At extreme conditions that prevail in most of the problems associated with shock waves, the assumption that the gas is ideal is no longer valid. Anisimov and Spiner [1] have taken an equation of state for non-ideal gases in a simplified form, and investigated the effect of the parameter for non-idealness on the problem of strong point explosions, which describes the behaviour of the medium satisfactorily at low densities. Ranga Rao and Purohit [19] have analysed the self-similar flow of a non-ideal gas driven by an expanding piston and obtained solutions by taking the equation of state suggested by Anisimov and Spiner [1].

The purpose of this study is to obtain self-similar solutions for the propagation of a shock wave in a dusty gas (a mixture of a non-ideal gas and small solid particles) with heat conduction and radiation heat flux, driven out by a piston moving with time according to exponential law. In order to obtain some essential features of the shock propagation, small solid particles are considered as a pseudo-fluid, and the mixture at a velocity and temperature equilibrium with a constant ratio of specific heats (Pai [15]). The motion of the piston is assumed to obey the law (Ranga Rao and Ramana [20]; Singh and Vishwakarma [25]; Vishwakarma and Nath [28])

$$r_p = A\exp(\mu t), \mu > 0 \qquad ...(1)$$

where r_p is the radius of the piston, A and μ are dimensional constants, and t is the time. 'A' represents initial radius of the piston.

Since we have assumed self-similarity, the shock will also be exponential. Therefore,

$$R = B\exp(\mu t), \qquad ....(2)$$

where R is the radius of the shock and B is a dimensional constant which is to be determined.

2. Fundamental Equations and Boundary Conditions

We consider the medium to be a dusty gas which is a mixture of small solid particles and a non-ideal gas. The equation of state of the non-ideal gas in the mixture is taken to be (Anisimov and Spiner [1], Ranga Rao and Purohit [19], Vishwakarma and Nath [29])

$$p_g = R^*\bar{\rho}_g(1 + b\bar{\rho}_g)T, \qquad ....(3)$$

where ρ_g and $\bar{\rho}_g$ are the partial pressure and partial density of the gas in the mixture, T is the temperature of the gas (and of the solid particles as the equilibrium flow condition is maintained), R^* is the specific gas constant and b is the internal volume of the molecules of the gas. In this equation, the deviations of an actual gas from the ideal state are taken into account, which result from the interaction between its component molecules. It is assumed that the gas is still so rarefied that triple, quadruple, etc., collisions between molecules are negligible, and their interaction is assumed to occur only through binary collisions.

The equation of state of the solid particles in the mixture is, simply,

$$\rho_{sp} = \text{constant},$$

where ρ_{sp} is the species density of the solid particles. Proceeding on the same lines as Pai [15], we obtain the equation of state of the mixture as

$$p = \frac{(1-k_p)}{(1-Z)}\{1 + b\rho(1-k_p)\}\rho R^* T, \qquad \text{.... (5)}$$

where p and ρ are the pressure and density of the mixture, $Z = V_{sp}/V$ is the volume fraction and $k_p = m_{sp}/m$ is the mass fraction (concentration) of the solid particles in the mixture, m_{sp} and V_{sp} being, respectively, the total mass and the volumetric extensions of the solid particles in a volume V and mass m of the mixture.

The relation between k_p and Z is given by (Pai [15])

$$k_p = \frac{Z\rho_{sp}}{\rho}.$$

In the equilibrium flow, k_p is constant in the whole flow-field. Therefore, from equation (6) Z/ρ = constant.

Also, we have the relation (Pai [15])

$$Z = \frac{k_p}{(1-k_p)G + k_p},$$

where $G = \rho_{sp}/\rho_g$ is the ratio of density of solid particles to the species density of the gas.

The internal energy per unit mass of the mixture may be written as

$$U_m = [k_p C_{sp} + (1-k_p)C_v]T = C_{vm}T, \qquad \text{....(9)}$$

where C_{sp} is the specific heat of the solid particles, C_v is the specific heat of the gas at constant volume and C_{vm} is the specific heat of the mixture at constant volume.

The specific heat of the mixture at constant pressure is

$$C_{pm} = k_p C_{sp} + (1-k_p)C_p], \qquad \text{.... (10)}$$

where C_p is the specific heat of the gas at constant pressure.

The ratio of the specific heats of the mixture is given by (Pai [15], Marble [10])

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \gamma\frac{(1+\delta\beta'/\gamma)}{(1+\delta\beta')}, \qquad \text{.... (11)}$$

where $\gamma = \frac{C_p}{C_v}$, $\delta = \frac{k_p}{(1-k_p)}$ and $\beta' = \frac{C_{sp}}{C_v}$

Now, $C_{pm} - C_{vm} = (1-k_p)(C_p - C_v) = (1-k_p)R^*,$ ....(12)

neglecting the term containing $b^2\rho^2$ (Anisimov and Spiner [1], Vishwakarma and Nath [30]).

The internal energy per unit mass of the mixture is, therefore, given by

$$U_m = \frac{p(1-Z)}{\rho(\Gamma-1)[1+b\rho(1-k_p)} \qquad \text{....(13)}$$

The fundamental equations for one-dimensional, unsteady and adiabatic flow of a mixture of a non-ideal gas and small solid particles with heat conduction and radiation heat flux taken into account, may be expressed as (Ghoniem et al [4]; Vishwakarma [27] ;Vishwakarma et al [32])

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{jpu}{r} = 0, \qquad \text{...(14)}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \qquad \text{....(15)}$$

$$\frac{\partial U_m}{\partial t} + u\frac{\partial U_m}{\partial r} - \frac{p}{\rho^2}\left(\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r}\right) + \frac{1}{pr^j}\frac{\partial}{\partial r}(r^j q) \qquad \text{...(16)}$$

where r and t are independent space and time coordinates, u is the flow velocity, q is the heat flux and $j = 1$ or 2 for cylindrical or spherical symmetry.

The total heat-flux q, which appears in the energy equation, may be decomposed as

$$q = q_C + q_R \qquad ...(17)$$

where $q_C =$ conduction heat flux and q_R = radiation heat flux.

According to Fourier's law of heat conduction,

$$q_C = -k\frac{\partial T}{\partial r}, \qquad ....(18)$$

where k is the coefficient of thermal conductivity and T is the absolute temperature.

Assuming local thermodynamic equilibrium and using the radiative diffusion model for an optically thick grey gas (Pomraning [17]), the radiative heat flux q_R may be obtained from the differential approximation of the radiation-transport equation in the diffusion limit as

$$q_C = -\frac{4}{3}\left(\frac{\sigma}{\alpha_R}\right)\frac{\partial T}{\partial r}, \qquad ....(19)$$

where σ is the Stefan-Boltzman constant and α_R is the Rosseland mean absorption coefficient.

The above mentioned thermal conductivity k and the absorption coefficient α_R are assumed to vary with temperature and density. These can be written in the form of power laws, namely (Ghoniem et al [4], Vishwakarma et al [32])

$$k = k_0\left(\frac{T}{T_0}\right)^{\beta_C}\left(\frac{\rho}{\rho_0}\right)^{\delta_C}, \alpha_R = \alpha_{R_0}\left(\frac{T}{T_0}\right)^{\beta_R}\left(\frac{\rho}{\rho_0}\right)^{\delta_C}, \qquad ....(20)$$

where subscripts 0 denote a reference state. The exponents in the above equations should satisfy the similarity requirements, if a self-similar solution is sought.

A shock is supposed to be propagating in the mixture of a non-ideal gas and small solid particles with constant density ρ_1 and constant pressure p_1 at rest (u_1=0).

The disturbance is headed by an isothermal shock (the formation of isothermal shock is a result of the mathematical approximation in which the flux is taken to be proportional to the temperature gradient; this excludes the possibility of a temperature jump, see for example Zel'dovich and Raizer [35]; Rosenau and Frankenthal [21, 22]; Vishwakarma and Nath [31]) and hence the conditions across it are

$$\begin{aligned}
&\rho_2(V-u_2)=\rho_1 V=m_s, \text{ (say)},\\
&p_2+\rho_2(V-u_2)^2=p_1+\rho_1 V^2,\\
&U_{m_2}+\frac{p_2}{\rho_2}+\frac{1}{2}(V-u_2)^2-\frac{q_2}{m_s}-U_{m_1}+\frac{p_1}{\rho_1}+\frac{1}{2}V^2,\\
&\frac{Z_2}{\rho_2}=\frac{Z_1}{\rho_1},\\
&T_2=T_1,
\end{aligned} \quad \text{....(21)}$$

where the suffices '1' and '2' refer to the values just ahead and just behind the shock front, respectively, $V=dR/dt$ denotes the velocity of the shock front. From equations (21), we obtain

$$u_z=(1-\beta)V,\ \rho_z=\frac{\rho_1}{\beta},\ \rho_z=\left[(1-\beta)+\frac{1}{\gamma M^2}\right]\rho_1 V^2, \quad \text{....(22)}$$

$$q_z=(1-\beta)\left[\frac{Z_1+\bar{b}(1-k_p)}{\lambda M^2(\beta-Z_1)[1+\bar{b}(1-k_p)}-\frac{1}{2}(1+\beta)\right]\rho_1 V^2,\ Z_z=\frac{Z_1}{\beta},$$

where $M=\left(\frac{\rho_1 V^2}{\gamma p_1}\right)^{1/2}$ is the shock Mach number referred to the frozen speed of sound $\left(\frac{\gamma p_1}{\rho_1}\right)^{1/2}$ and $\bar{b}=b\rho_1$ is the parameter of non-idealness of the gas. The quantity $\beta(0<\beta<1)$ is obtained by the relation

$$\beta^2-\left[Z_1+1+\frac{1}{\gamma M^2}\right]\beta^2+\left[\frac{Z_1\bar{b}(1-k_p)(1+\gamma M^2)+Z_1\gamma M^2+1}{\gamma M^2\{1+\bar{b}(1-k_p)\}}\right]\beta$$

$$+\frac{(1-Z_1)\bar{b}(1-k_p)}{\gamma M^2\{1+\bar{b}(1-k_p)\}}=0 \quad \text{...(23)}$$

The expression for the initial volume fraction of the solid particles Z_1 is given by, from equation (8),

$$Z_1 = \frac{V_{sp}}{V_1} = \frac{k_p}{(1-k_p)G_1 + k_p} \quad \text{....(24)}$$

where $G_1 = \rho_{sp} / \rho g_1$ is the ratio of the species density of solid particles to the initial species density of the gas in the mixture.

3. Similarity solutions

To obtain similarity solutions, we write the unknown variables in the following form

$$u = VU(\eta), \quad \rho = \rho_1 D(\eta), \quad p = \rho_1 V^2 P(\eta), \quad q = \rho_1 V^2 Q(\eta), \quad Z = Z_1 D(\eta), \quad \text{....(25)}$$

where *U, D, P* and *Q* are functions of the non-dimensional variable (similarity variable) $\eta = r/R$ only. The variable η assumes the value '1' at the shock front and η_p on the piston.

Equations (1), (2) and (25) yield a relation between A and B in the form

$$A = B\eta_p . \quad \text{....(26)}$$

By use of equations (25), the conservation equations (14) to (16) can be transformed into a system of ordinary differential equations with respect to η

$$(U-\eta)\frac{dD}{d\eta} + D\frac{dU}{d\eta} + \frac{jDU}{\eta} = 0, \quad \text{....(27)}$$

$$(U-\eta)\frac{dU}{d\eta} + D + \frac{1}{D}\frac{dP}{d\eta} = 0, \quad \text{....(28)}$$

$$\frac{dP}{d\eta} - T\frac{dD}{d\eta} + \frac{2P}{(U-\eta)} + H\left[\frac{dQ}{d\eta} + \frac{jQ}{\eta}\right] = 0, \quad \text{....(29)}$$

where

$$T = \frac{PL}{D}, \quad H = \frac{(\Gamma - 1)\{1 + \bar{b}D(1-k_p)\}}{(1 - Z_1 D)(U-\eta)},$$

$$L = \frac{1 + 2\bar{b}D(1-k_p) - \bar{b}Z_1 D^2(1-k_p) + (\Gamma - 1)\{1 + \bar{b}D(1-k_p)\}}{\{(1+\bar{b}D)(1-k_p)\} - (1 - Z_1 D)}$$

By using equations (18)-(20) in (17), we obtain

$$q = -\left[\frac{k_0}{T_0^{\beta_c}\rho_0^{\delta_c}} T^{\beta_c}\rho^{\delta_c} + \frac{16}{3}\frac{\sigma}{\alpha_{R_n}} T_0^{\beta_R}\rho_0^{\delta_R} T^{3-B_R}\rho^{-\delta_R}\right]\frac{\partial T}{\partial t}. \quad ....(30)$$

Using equations (5) and (25) in (30), we obtain

$$Q = -\left[\frac{k_0 \rho_1^{\delta_c - 1}\mu}{T_0^{\beta_c}\rho_0^{\delta_c}(1-k_p)^{1+\beta_c} R^{*4-\beta_c}} \frac{V^{2\beta_c - 2} P^{\beta_c} D^{\delta_c - \beta_c}(1 - Z_1 D)^{\beta_c}}{\{1 + \bar{b}D)(1-k_p)\}^{\beta_c}} + \frac{16\sigma T_0^{\beta_R}\rho_0^{\delta_R}\mu\rho_1^{-\delta_R - 1}}{3\alpha_{R_0}(1-k_p)^{4+\beta_R} R^{*4-\beta_R}} \frac{V^{4-2\beta_R} P^{3-\beta_R} D^{\beta_R - \delta_R - 3}(1 - Z_1 D)^{3-\beta_R}}{\{1 + \bar{b}D)(1-k_p)\}^{3-\beta_R}}\right]$$

$$\frac{d}{d\eta}\frac{P(1 - Z_1 D)}{[D + \bar{b}D^2(1-k_p)]} \quad ....(31)$$

Equation (31) shows that the similarity solution of the present problem exist only when $\beta_c = 1$ and $\beta_R = 2$.

Therefore equation (31) becomes

$$Q = -X\left[\frac{1 - Z_1 D}{D + \bar{b}D^2(1-k_p)}\frac{dP}{d\eta} + \frac{P(Z_1 D - 2)\bar{b}D(1-k_p) - PdD}{\{D + \bar{b}D^2(1-k_p)\}^2}\right], \quad ....(32)$$

where $X = \left[\Gamma_c D^{\delta_c - 1} + \Gamma_R D^{-\delta_R - 1}\right]\left[\frac{P(1 - Z_1 d)}{[1 + \bar{b}D(1-k_p)}\right]$ and Γ_c and Γ_R are the conductive and radiative heat transfer parameters, respectively and they are given by

$$\Gamma_c = \frac{k_0 \rho_1^{\delta_c - 1}\mu}{T_0 \rho_0^{\delta_c}(1-k_p)^2 R^{*2}} \text{ and } \Gamma_R = \frac{16\sigma T_0^2 \rho_0^{\delta_R}\mu\rho_1^{-\delta_R - 1}}{3\alpha_{R_0}(1-k_p)^2 R^{*2}}.$$

By solving equations (27)-(29) and (32) for $\frac{dU}{d\eta}, \frac{dP}{d\eta}, \frac{dQ}{d\eta}$ and $\frac{dD}{d\eta}$, we have

$$\frac{dU}{d\eta} = -\frac{(U - \eta)}{D}\frac{dD}{d\eta} - \frac{jU}{\eta}, \quad ...(33)$$

$$\frac{dP}{d\eta}=(U-\eta)^2\frac{dD}{d\eta}+(U-\eta)\frac{jDU}{\eta}-UD, \qquad ....(34)$$

$$\frac{dQ}{d\eta}=\frac{1}{H}\left[\{(U-\eta)^2-T\}\frac{dD}{d\eta}+(U-\eta)\frac{jDU}{\eta}-UD+\frac{2P}{U-\eta}+\frac{jQH}{\eta}\right], \qquad ...(35)$$

$$\frac{dD}{d\eta}=\left[(1-Z_1D)U-\frac{(1-Z_1D)(U-\eta)U}{\eta}-\frac{Q\{1+\bar{b}D(1-k_p\_}{X}\right]\times$$

$$\left[\frac{D^2\{1-\bar{b}D(1-k_p)\}}{(1-Z_1D)D\{1+\bar{b}D(1-k_p)\{(U-\eta)^2-P+P\bar{b}D^2(1-k_p)Z_1D-2}\right] \qquad ...(36)$$

Using the similarity transformations (25), equations (22) can be rewritten as

$$U(1)=(1-\beta),\quad D(1)=\frac{1}{\beta},\quad P(1)=\frac{1}{\gamma M^2}+(1-\beta),$$

$$Q(1)=(1-\beta)\left[\frac{Z_1+\bar{b}(1-k_p)}{\gamma M^2(\beta-Z_1)\{1+\bar{b}(1-k_p)\}}-\frac{1}{2}(1+\beta)\right] \qquad ....(37)$$

In addition to the shock conditions (37), the kinematic condition at the piston surface, which in the non-dimensional form is

$$U_{(\eta_p)}=\eta_p, \qquad ....(38)$$

must be satisfied.

For exhibiting the numerical solutions, it is convenient to write the field variables in non-dimensional form as

$$\frac{u}{u_2}=\frac{U(\eta)}{U(1)},\ \frac{\rho}{\rho_2}=\frac{D(\eta)}{D(1)},\ \frac{p}{p_2}=\frac{P(\eta)}{P(1)},\ \frac{q}{q_2}=\frac{Q(\eta)}{Q(1)}. \qquad ....(38)$$

4. Results and discussion

Distribution of the flow variables between the shock surface (η=1) and the piston ($\eta=\eta_p$) are obtained by numerical integration of equations (33) to (36) with boundary conditions (37). For the purpose of numerical calculations, the values of the constant parameters are taken to be (Ghoniem et al [4]; Miura and Glass [13];

Pai et al [16]; Vishwakarma [27]; Vishwakarma and Nath [29]) $j = 2$; $\gamma = 1.4$; $k_p = 0$, 0.02, 0.4; $G_1 = 50$, 100; $\beta' = 1$; $\bar{b} = 0, 0.05$; $M^2 = 25$; $\delta_c = 1$; $\delta_R = 2$; $\Gamma_c = 10$; $\Gamma_R = 100$. The values $\gamma = 1.4$, $\beta' = 1$ may correspond to the mixture of air and glass particles (Miura and Glass [13]). The value $j = 2$ corresponds to spherical shock, the value $k_p = 0$, corresponds to the dust free case and $k_p = 0$, $\bar{b} = 0$ to the perfect gas case. The value M =5 of the shock Mach number is appropriate, because we have treated the flow of a mixture of a non-ideal gas and a pseudo-fluid (small solid particles) at a velocity and temperature equilibrium. The set of values $\delta_c = 1$, $\delta_R = 2$ is representative of the case of a high-temperature, low density medium (Ghoniem et al [4]). Also, the set of values $\Gamma_c = 10, \Gamma_R = 100$ is representative of the case in which there is heat transfer by both conduction and radiative diffusion.

Values of the density ratio across the shock front β and the piston position η_p are shown in Table 1 for different values of $\bar{b}, k_p$ and G_1 with $\gamma = 1.4, M^2 = 25$, $\delta_c = 1, \delta_R = 2, \Gamma_c = 10, \Gamma_R = 100$ and $\beta' = 1$.

Figures 1 to 4 show the variation of the flow variables $\frac{u}{u_2}, \frac{\rho}{\rho_2}, \frac{p}{p_2}, \frac{q}{q_2}$ with η at various values of the parameters $\bar{b}, k_p$ and G_1. It is shown that, as we move inward from the shock front towards the inner expanding surface, the reduced velocity u/u_2, the reduced density ρ/ρ_2 and the reduced pressure p/p_2 increase whereas the reduced total heat flux q/q_2 decreases. The behaviour of the heat flux profiles is similar to those obtained by Elliott [3], Ghoniem et al[4] and Vishwakarma et al [32].

Table 1. Density ratio $\beta = \left(\frac{\rho_1}{\rho_2}\right)$ across the shock front and the position of the piston surface η_p for different values of $\bar{b}, k_p$ and G_1 with $\gamma = 1.4, M^2 = 25, \delta_c = 1$, $\delta_R = 2, \Gamma_c = 10, \Gamma_R = 100$ and $\beta' = 1$.

$\bar{b}$	k_p	G_1	β	η_p
0	0	-	0.0285714	0.991758
	0.2	50	0.0335466	0.990148
		100	0.0310652	0.990978
	0.4	50	0.0417293	0.987421
		100	0.0351939	0.989578
0.05	0	-	0.0538411	0.983888
	0.2	50	0.0538513	0.983814
		100	0.0521034	0.984340
	0.4	50	0.0563127	0.982880
		100	0.0513063	0.984517
0.1	0	-	0.0672148	0.979907
	0.2	50	0.0656324	0.980261
		100	0.064034	0.980763
	0.4	50	0.0659347	0.979999
		100	0.0613713	0.981556

It is found that the effects of an increase in the value of the parameter of non-idealness $\bar{b}$ of the gas are

(i) to increase the value of β (i.e. to decrease the shock strength, see Table 1);

(ii) to increase the distance of the piston from the shock front(see Table 1), i.e. the flow field behind the shock becomes somewhat rarefied. This shows the same result as in (i), i.e. there is a decrease in the shock strength;

(iii) to increase the velocity u/u_2 and the total heat flux q/q_2, and to decrease the density ρ/ρ_2 and the pressure p/p_2 at any point in the flow field behind the shock (see figures 1-4).

In fact, the non-idealness of the gas lowers the compressibility of the medium, which results in (i),(ii) and (iii) given above.

The effects of an increase in the ratio of density of solid particles to the initial density of gas G_1 are:

(i) to decrease β (i.e. to increase the shock strength, see Table 1);

(ii) to decrease the distance between the piston and the shock front (see Table 1). This means that an increase in the ratio of the density of solid particles to the initial density of the gas has an effect of increasing the shock strength, which is the same as indicated in (i) above;

(iii) to decrease the velocity u/u_2 and the total heat flux q/q_2 but to increase the density ρ/ρ_2 and the pressure p/p_2 (see figures 1-4).

The above effects are more impressive at higher values of k_p. These effects may be physically interpreted as follows:

By an increase in G_1 (at constant k_p), there is a high decrease in Z_1, i.e. the volume fraction of solid particles in the undisturbed medium becomes, comparatively, very small. This causes comparatively more compression of the mixture in the region between the shock and the piston, which displays the above effects.

The effects of an increase in the mass concentration of the solid particles k_p are:

(i) to decrease the shock strength (to increase the value of β) when $G_1 = 50$, and to increase it, when $G_1 = 100$ except in the case when $\bar{b} = 0$;

(ii) to increase the distance of piston from the shock front when $G_1 = 50$ and to decrease it when $G_1 = 100$ except in the case when $\bar{b} = 0$;

(iii) to decrease the velocity u/u_2, the density ρ/ρ_2 and the total heat flux q/q_2 at any point in the flow field behind the shock front(see figures 1,2, and 4);

(iv) to decrease the pressure p/p_2 when $G_1 = 50$, and to increase it when $G_1 = 100$ except in the case when $\bar{b} = 0$.

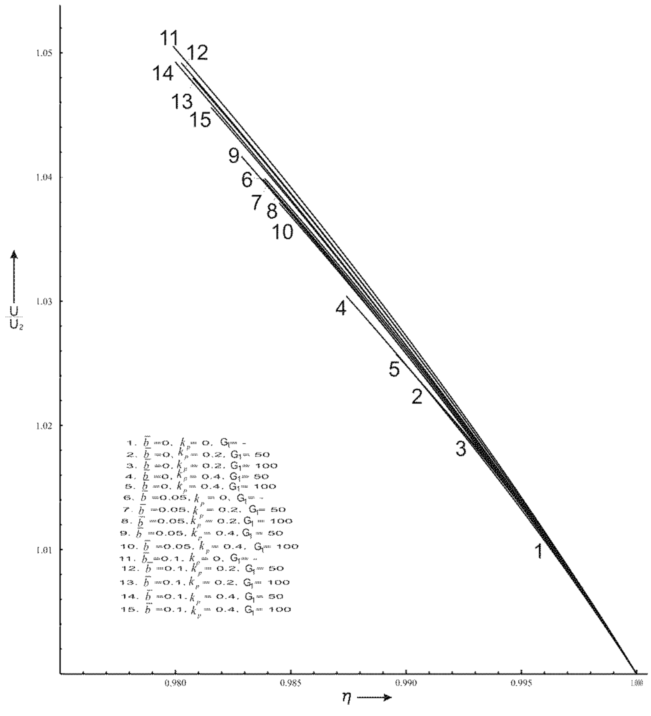

Fig.1. Variation of the reduced velocity $\frac{U}{U_2}$ in the flow-field behind the shock front

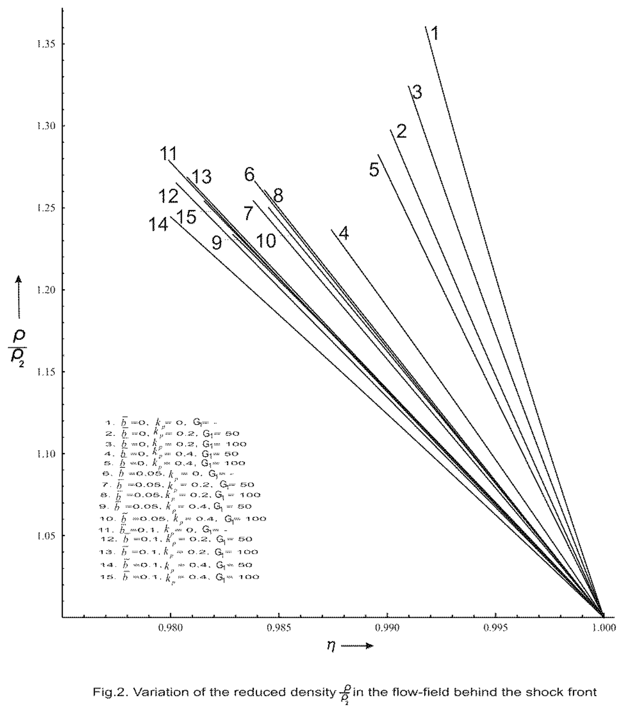

Fig.2. Variation of the reduced density $\frac{\rho}{\rho_2}$ in the flow-field behind the shock front

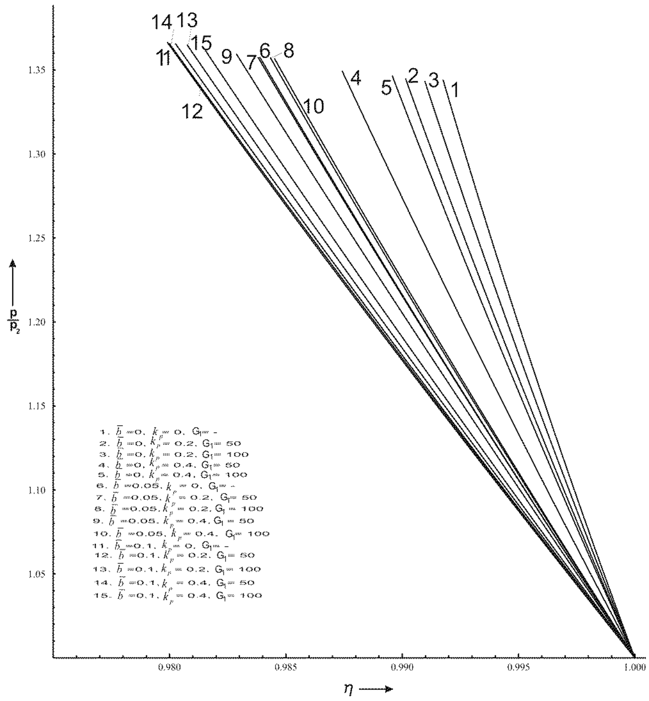

Fig.3. Variation of the reduced pressure $\frac{p}{p_2}$ in the flow-field behind the shock front

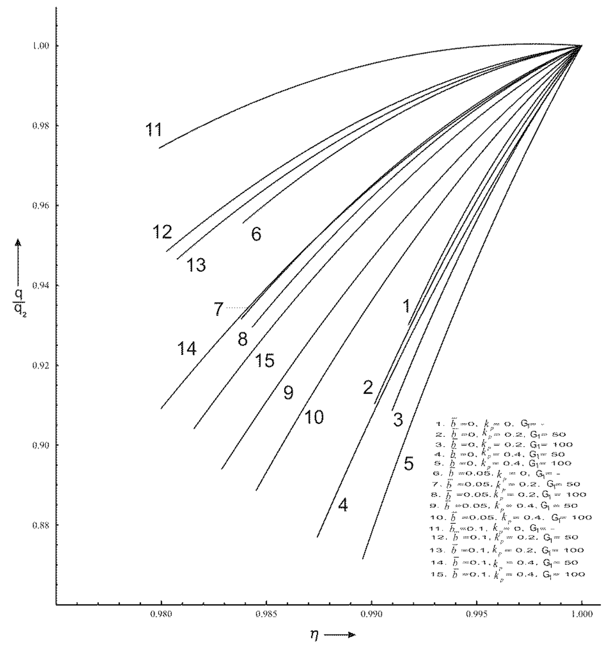

Fig.4. Variation of the reduced total heat flux $\frac{q}{q_2}$ in the flow-field behind the shock front

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