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LRS BIANCHI TYPE – I STRING COSMOLOGICAL MODELS WITH DECAYING VACUUM ENERGY

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Abstract : LRS (Locally Rotationally Symmetric) Bianchi Type I string models (string dust and massive) are investigated. To get the deterministic model of the universe, we assume that the shear (σ) is proportional to expansion (θ) and vacuum energy (Λ) α H^2, H being mean Hubble parameter. The reality condition $\rho > 0$ are satisfied for both the models. In both the models, big bang scenario exists. Both the models represent decelerating and accelerating phases which match with the astronomical observations (Ries et al. [15]). The anisotropy is maintained throughout in both the models but the models isotropize in special case. Particle horizon exists in both the models. The physical and geometrical aspects of models are also discussed.

Keywords : LRS Bianchi - I, String dust, Vacuum Energy

2010 Mathematics Subject Classification : 83CXX, 83 FXX

1. Introduction

Astronomical observations indicate that initial stage of universe is anisotropic which represents isotropic and homogeneous universe in the present day. The Friedman-Robertson-Walker (FRW) models which are homogeneous and isotropic are considered the best model to represent the present day universe. But

FRW models are unstable near the singularity, therefore anisotropic and homogeneous Bianchi models are undertaken into the study. Bianchi Type I models are helpful to understand the universe in its early stage of evolution of universe. Cosmic strings play a significant role in structure formation and evolution of universe as investigated by (Kibble [11]). These strings have stress energy and classified as massive and geometrical strings (Nambu string). The pioneer work in the formation of energy-momentum tensor for classical massive strings is due to Letelier [12]. He explained that massive strings are formed by geometric strings (Stachel [17]) with particle attached along its extension. In this regard, Banerjee et al. [6], Tikekar and Patel [18], Roy and Banerjee [16]. Bali et al. [2,3,4] investigated string cosmological models using the form of energy-momentum tensor as given by Letelier [12] in different contexts.

A wide range of astronomical observations suggest that universe possesses a non-zero cosmological constant which is considered the most favoured candidate of dark energy representing energy density of vacuum. Zel'dovich [19] and Dreitlein [10] studied its significance. Barrow and Shaw [7] suggested that cosmological term corresponds to a very small value of the order 10^{-122} when applied to Friedmann model. A number of cosmological models in which cosmological constant decays with time have been investigated by Bertolami [9], Berman [8], Bali and Singh [5], Ram and Verma [14].

In this paper, LRS Bianchi Type I string cosmological models (string dust and massive string) are investigated. For deterministic models of universe, we have assumed two conditions (i) $\sigma \propto \theta$ (ii) $\Lambda \propto H^2$. The physical and geometrical aspects of the models related with the observations are also discussed.

2. The metric and field equations

We consider LRS (Locally Rotationally Symmetric) space-time as

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 (dy^2 + dz^2) \qquad \ldots(1)$$

where A and B are metric potentials and are functions of t-alone.

The Einstein's field equation with decaying vacuum energy (Λ) is given by

$$R_i^j - \frac{1}{2} R\, g_i^j = -\, T_i^j + \Lambda\, g_i^j \qquad \ldots(2)$$

(in geometrized units $8\pi G = 1, c = 1$)

The energy-momentum tensor (T_i^j) for string dust and massive string is given by Letelier [12] as

$$T_i^j = \rho\, v_i\, v^j - \lambda\, x_i\, x^j \qquad \ldots(3)$$

where $\rho = \rho_p + \lambda, \rho$ being rest energy density, ρ_p the particle density, λ the string tension density. We assume the coordinates to be comoving so that

$$v_1 = 0 = v_2 = v_3, v_4 = -1, v^4 = 1.$$

Here x^i represents direction of string satisfying

$$v_i\, v^i = -x_i\, x^i = -1 \qquad \ldots(4)$$

$$v_i x^i = 0 \qquad \ldots(5)$$

We assume the direction of string along x-axis so that $x_1\, x^1 = 1$.

Now the field equation (2) for the metric (1) leads to

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} = \lambda + \Lambda \qquad \ldots(6)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = \Lambda \qquad \ldots(7)$$

$$\frac{2A_4 B_4}{AB} + \frac{B_4^2}{B^2} = \rho + \Lambda \qquad \ldots(8)$$

For massive string (Letelier [12]), we have

$$\rho = \rho_p + \lambda$$

Case (i): String dust model

For string dust, $\rho_p = 0$. Thus, we have

$$\rho = \lambda \qquad \ldots(9)$$

3. Solutions of field equations

From (6) and (7), we have

$$\frac{A_{44}}{A}-\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{B_4^2}{B^2}=-\lambda \qquad \ldots(10)$$

From equations (8) and (10) and using $\rho = \lambda$, we have

$$\frac{A_{44}}{A}-\frac{B_{44}}{B}+\frac{3A_4B_4}{AB}=\Lambda \qquad \ldots(11)$$

To get the deterministic solution of equation (11), we assume that

$$\sigma \, \alpha \, \theta \qquad \ldots(12)$$

and the decaying vacuum energy (Λ) is proportional to H^2 (Arbab [1]) as

$$\Lambda \, \alpha \, H^2 \qquad \ldots(13)$$

where σ is shear, θ the expansion, H the Hubble parameter.

From (12) and(13), we have

$$A = B^{2n} \qquad \ldots(14)$$

and $$\Lambda=\frac{\beta}{9}\left(\frac{A_4}{A}+\frac{2B_4}{B}\right)^2 \qquad \ldots(15)$$

Using (15) in (11), we have

$$\frac{A_{44}}{A}-\frac{B_{44}}{B}+\frac{3A_4B_4}{AB}=\frac{\beta}{9}\left(\frac{A_4^2}{A^2}+\frac{4B_4^2}{B^2}+\frac{4A_4B_4}{AB}\right) \qquad \ldots(16)$$

which leads to

$$\frac{B_{44}}{B}=\frac{(n+1)}{9(2n-1)}[4\beta(n+1)-36n]\frac{B_4^2}{B^2}$$

Thus, we have

$$\frac{B_{44}}{B}=\gamma\frac{B_4}{B} \qquad \ldots(17)$$

where $\gamma = \frac{(n+1)}{9(2n-1)}[4\beta(n+1)-36n]$ ...(18)

Equation (17) leads to

$$B^{1-\gamma} = (at+b) \qquad \text{...(19)}$$

Thus

$$B^2 = (at+b)^{\frac{2}{1-\gamma}} \qquad \text{...(20)}$$

and $A^2 = B^{4n} = (at+b)^{\frac{4n}{1-\gamma}}$ ...(21)

Thus the metric (1) leads to

$$ds^2 = -dt^2 + (at+b)^{\frac{4n}{1-\gamma}} dx^2 + (at+b)^{\frac{2}{1-\gamma}}(dy^2+dz^2) \qquad \text{...(22)}$$

Case (ii) : Massive string

For massive string, we have

$$\rho = \rho_p + \lambda \qquad \text{...(23)}$$

For deterministic solution, we use the condition $\sigma \alpha \theta$ *and* $\Lambda \alpha H^2$.

Thus, we have

$$A = B^{2n} \qquad \text{...(24)}$$

and $\Lambda = \frac{\beta}{9}\left(\frac{A_4}{A} + \frac{2B_4}{B}\right)^2$ ...(25)

where β is constant.

Equations (24), (25) and (7) lead to

$$(2n+1)\frac{B_{44}}{B} = \left[\frac{4\beta(n+1)^2 - 36n^2}{9}\right]\frac{B_4^2}{B^2} \qquad \text{...(26)}$$

which leads to

$$\frac{B_{44}}{B} = s\,\frac{B_4}{B} \qquad \ldots(27)$$

where $$s = \frac{4\beta(n+1)^2 - 36n^2}{9(2n+1)} \qquad \ldots(28)$$

Equation (27) leads to

$$B = (\ell t + m)^{1/1-s} \qquad \ldots(29)$$

where ℓ and m are constants of integration. Therefore,

$$B^2 = (\ell t + m)^{\frac{2}{1-s}} \qquad \ldots(30)$$

and $$A^2 = B^{4n} = (\ell t + m)^{\frac{4n}{1-s}} \qquad \ldots(31)$$

Now the metric leads to

$$ds^2 = -dt^2 + (\ell t + m)^{\frac{4n}{1-s}} dx^2 + (\ell t + m)^{\frac{2}{1-s}} (dy^2 + dz^2) \qquad \ldots(32)$$

4. Physical and geometrical features

String dust model

The rest energy density (ρ), the string tension (λ), the expansion (θ), the shear (σ), the Hubble parameter (H), the spatial volume (R^3), deceleration parameter (q) and particle horizon for the model (22) are given by

$$\rho = \frac{a^2}{9(1-\gamma)^2 (at+b)^2}[9(4n+1) - 4(n+1)^2]$$

$$= \lambda \qquad \ldots(33)$$

$$\theta = \frac{2(n+1)a}{(1-\gamma)(at+b)} \qquad \ldots(34)$$

$$\sigma = \frac{1}{\sqrt{3}}\frac{(2n-1)a}{(1-\gamma)(at+b)} \qquad \ldots(35)$$

$$H = \frac{1}{3}\theta$$

$$= \frac{2(n+1)a}{3(1-\gamma)(at+b)} \qquad \ldots(36)$$

$$R^3 = (at+b)\frac{2(n+1)}{1-\gamma} \qquad \ldots(37)$$

$$q = -\frac{(2n-1)}{2(n+1)} \qquad \ldots(38)$$

$$\text{Particle Horizon} = \int_{t_0}^{t} \frac{dt}{R^3(t)}$$

$$= \int_{t_0}^{t} \frac{dt}{(at+b)^{\frac{2(n+1)}{1-\gamma}}} = \text{finite} \qquad \ldots(39)$$

Massive string model

The rest energy density (ρ), the string tension (λ), the particle density (ρ_p), the expansion (θ), the shear (σ), the mean Hubble parameter (H), the spatial volume (R^3), deceleration parameter (q) and particle horizon for the model (32) are given by

$$\rho = [9(4n+1) - 4\beta(n+1)^2]\frac{\ell^2}{(1-s)^2(\ell t+m)^2} \qquad \ldots(40)$$

$$\lambda = \frac{\ell^2}{9(1-s)^2(\ell t+m)^2}[18s + 9 - 4\beta(n+1)^2] \qquad \ldots(41)$$

$$\rho_p = \rho - \lambda = \frac{\ell^2}{9(1-s)^2(\ell t+m)^2}[81(4n+1) - 32\beta(n+1)^2 - 18s - 9] \qquad \ldots(42)$$

$$\theta = \frac{2(n+1)\ell}{(1-s)(\ell t+m)} \qquad \ldots(43)$$

$$\sigma = \frac{\ell(2n-1)}{\sqrt{3}(1-s)(\ell t+m)^2} \qquad \ldots(44)$$

$$H = \frac{2(n+1)\ell}{3(1-s)(\ell t+m)} \quad ...(45)$$

$$R^3 = AB^2 = B^{2n+2} = (\ell t+m)^{\frac{2(n+1)}{1-s}} \quad ...(46)$$

$$q = \frac{(n+1)\ell^2(4n+6s-2)}{9(1-s)^2(\ell t+m)^2} \quad ...(47)$$

Particle Horizon $= \int_{t_0}^{t} \frac{dt}{R^3(t)}$

$$= \int_{t_0}^{t} \frac{dt}{(\ell t+m)^{\frac{2(n+1)}{1-s}}} = \text{finite} \quad ...(48)$$

5. Conclusion

The model (22) for string dust starts with a big bang at $t = -b/a$ and the expansion in the model decreases as t increases and finally stops at $t = \infty$. The model in general represents anisotropic space-time and isotropizes for $n = \frac{1}{2}$ The Hubble parameter is initially large but decreases due to lapse of time. The spatially volume increases with time. The deceleration parameter $q < 0$ for $n > \frac{1}{2}$ and $q > 0$ for $n < \frac{1}{2}$. Thus the model represents decelerating and accelerating phases of universe which matches with the astronomical observation as investigated by Riess et al. (15). Equation (39) indicates that particle horizon exists in the model. The reality condition $\rho > 0$ is satisfied if $9(4n+1) > 4\beta(n+1)^2$.

There is Point Type singularity in the model at t = - b/a when n > 0, $1-\gamma > 0$. (MacCallum [19])

The reality condition $\rho > 0$ for the massive string model (32) is satisfied if $9(4n+1) > 4\beta(n+1)^2$.

The model starts with a big bang at $t = -\frac{m}{\ell}$ and the expansion in the model decreases as t increases and when $t \to \infty$ then the expansion stops. The energy density (ρ), string tension (λ) and particle density (ρ_p) are initially large but decreases due to lapse of time. The mean Hubble parameter is initially large but decreases as time passes. The spatial volume increases with time. The deceleration parameter $q > 0$ if $4n+6s-2 > 0$ and $n+1 > 0$ and $q < 0$ if $4n+6s - s > 0$ and $n+1 < 0$. Equation (48) shows that particle horizon exists in the model. There is Point Type singularity in the model at $t = -m/\ell$ when $n > 0$ and $1 - s > 0$. However, if $n < 0$ and $1 - s > 0$ then the model has Cigar type singularity (MacCallum [13]). The anisotropy is maintained in the model throughout but isotropizes in special case.

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