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THE STRESS-INTENSITY FACTORS DUE TO THREE GRIFFITH-CRACKS OPENED BY THERMAL STRESS IN AN INFINITE ORTHOTROPIC MEDIUM

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Abstract : The closed form expressions for stress-intensity factors and of crack opening displacement are obtained by using Fourier Transform Method. A special case of point heat source is discussed. The stress possess square root singularity at crack tips.

Keywords : Stress-intensity factors (SIF), crack opening displacement (COD), Fourier transform method (FTM), heat source (HS)

2010 Mathematics Subject Classification : 74A99; 42A48

1. Introduction

Composite materials are very commonly used now a days. They can easily be manipulated to desired weight-to-strength ratios. These can be made with less weight and more strength then steel. Therefore these materials are suitable in aerospace structures.

These composites are made by putting matrix and fibers alternatively. If the layers of fibers are more than six, then these composites can be treated as orthotropic materials. It have been proved experimentally by Bandopadhya et. al. [1] and theoretically by Sharma [14].

The problems of thermal stress has its beginning in 1833 when Duhamel derived equations for the distribution of strain in elastic medium containing temperature gradients. Duhamel's result where produced by a number of workers and specially in the hand of Neumann came to be expressed in the present form known as Duhamel – Neumann relations. For anisotropic medium see Lekhnitsikii [11].

There are no many problems for isotropic medium with one, two or more Griffith cracks opened by mechanical force [16],Parihar and Kushwaha [13]extended to isotropic strip with one or two Griffith cracks. Kushwaha [9] extended to orthotropic strip with one and two Griffith-cracks.

There are very few problems of Griffith-crack opened by thermal stress. Singh [15] had solved the problem of single and double Griffith-cracks opened by thermal stress in infinite orthotropic medium. Choi et.al. [4]solved for conduction of heat with debonding of composites. De and Patra [5] solved for cruciform crack in orthotropic plane. Li [12] has given a generalized theory of thermo-elasticity in anisotropic infinitemedium. Keeret. al. [8] calculated the disturbance in heat flow by a line crack in infinite anisotropic thermo-elastic solid.

Kardomateas [7] had solved for thermo-elasticity of filament of orthotropic elliptic cylinder. Chen [3] solved for thermo-elastic problem of an anti-symmetrical heat flow disturbed by three coplanar cracks.

In the present research paper three Griffith-cracksopened by thermal stress which is developed by heat source. The cracks occupy the region $y = 0$, $x \in 0 \le |x| < b,\ c < |x| < d$. We have assumed that co-ordinate axis coincide with axes of orthotropy. We have also assumed that the medium is under plane-strain conductions. It is also assumed that elastic properties of medium are not changed due to heat and the crack opening does not alter heat distribution.

The physical problem is reduced to the following mixed-boundary conditions, see figure-1,as

$$\sigma_{xy}(x,0) = 0, 0 \le |x| < \infty \qquad .... (1)$$

$$u_y(x,0) = 0, x \in I_2 + I_4 \qquad .... (2)$$

$$\sigma_{yy}(x,0) = 0, x \in I_1 + I_3 \qquad .... (3)$$

with

$$I_1 = I_1^+ + I_1^-,\ I_2 = I_2^+ + I_2^-,\ I_3 = I_3^+ + I_3^-,\ I_4 = I_5^+ + I_4^-$$

$$I_1^+ = [0, b),\ I_2^+ = (b, c),\ I_3^+ = [c, d),\ I_4^+ = [d, \infty), \qquad .... (4)$$

$$I_1^- = (-b, 0],\ I_2^- = (-c, -b),\ I_3^- = [-d, -c),\ I_4^- = (-\infty, -d]$$

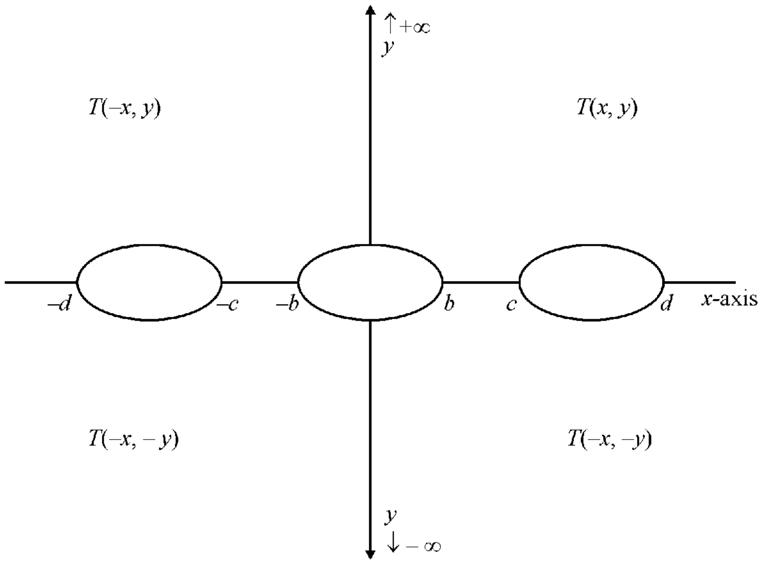

Figure 1. Three Griffith cracks opened by Temperature *T* developed by Heat source *Q*(*x*, *y*) in orthotropic infinite medium.

where σ_{xy}, σ_{yy}, and u_x, u_y are components of stress and of displacement at general point (x, y). We assume that normal stress and component u_y of displacement are taken as,

$$\sigma_{ij}(x, y) = \sigma_{ij}^{(e)}(x, y) + \sigma_{ij}^{(h)}(x, y),\ i, j = x, y$$

$$u_i(x, y) = u_i^{(e)}(x, y) + u_i^{(h)}(x, y),\ i = x, y$$

where superscripts (e) and (h) stand for elastic and heat part respectively.

The symmetry of geometry reduces the boundary conditions (1) – (3) to the following

$$\sigma_{xy}(x, 0) = 0, 0 \le x < \infty, \qquad .... (5)$$

$$u_y(x,0) = 0, x \in I_2^+ + I_4^+ \qquad \text{.... (6)}$$

$$\sigma_{yy}(x,0) = 0, x \in I_1^+ + I_3^+ \qquad \text{.... (7)}$$

where I_i^+, i = 1, 2, 3, 4 are given by (4). The Fourier transforms are defined as

$$f_{c,s}(p) = \int_0^\infty f(x)\{\cos px, \sin px\}dx$$

with inversion as

$$f(x)\frac{2}{\pi}\int_0^\infty f_{c,s}(p)\{\cos px, \sin px\}dp.$$

wechecked throughout that, see [2].

$$u_y^{(e)}(x,0) > 0, x \in I_1 + I_3,$$

which means that cracks, really open out and crack faces do not meet each other except at crack tips. The plan of the paper is as follows :In section 2we shall formulate heat problem. Section 3 will solve elasticity problem. The reduction to and solution of quintuple integral equation will be done in section 4.The general expressions of physical quantities will be reported in section 5. A special case of point heat source is discussed in section 6. The discussion and conclusion of research endeavour is given in section 7.

2. Formulation of heat problem

The formulation of heat problem is done by taking appropriate Fourier sine and cosine transforms, with respect to x and y variables, of equations of equilibrium, in the absence of body forces,

$$\frac{\partial\sigma_{xx}^{(h)}}{\partial x} + \frac{\partial\sigma_{xy}^{(h)}}{\partial y} = 0, \frac{\partial\sigma_{xy}^{(h)}}{\partial x} + \frac{\partial\sigma_{yy}^{(h)}}{\partial y} = 0$$

and then using the values of Fourier transforms of σ_{xx}, σ_{xy}, σ_{yy} obtained from the stress-strain relations

$$\left.\begin{aligned}\sigma_{xx}^{(h)} &= \frac{a_{22}e_x^{(h)} - a_{12}e_y^{(h)}}{\alpha} - k_1 T \\ \sigma_{xy}^{(h)} &= e_{xy}^{(h)} / a_{66} \\ \sigma_{yy}^{(h)} &= \frac{(a_{22}e_y^{(h)} - a_{12}e_x^{(h)})}{\alpha} - k_2 T\end{aligned}\right\} \quad .... (8)$$

where

$$\alpha = a_{11}a_{22} - a_{12}^2, \quad k_1 = \alpha_{t_1}(a_{11} - a_{12})a_{22}, \quad k_2 = \alpha_{t_2}(a_{22} - a_{12})a_{11}$$

and α_{t_1}, α_{t_2} are the coefficients of linear expansion and $a_{11} \sim a_{66}$ are elastic constants of the medium. $e_x^{(h)}$, $e_y^{(h)}$ and $e_{xy}^{(h)}$ are components of strain. We shall get two linear algebraic equations in $u_{x_{sc}}^{(h)}$, $u_{y_{cs}}^{(h)}$. Solving for $u_{x_{sc}}^{(h)}$ and $u_{y_{cs}}^{(h)}$ respectively and then inverting we get as,

$$u_x^{(h)}(x, y) = \frac{4}{\pi^2}\int_0^\infty\int_0^\infty T_{cc}(p, q)\sin(px)\cos(qy)W_1\,dp\,dq \quad .... (9)$$

$$u_x^{(h)}(x, y) = \frac{4}{\pi^2}\int_0^\infty\int_0^\infty T_{cc}(p, q)\cos(px)\sin(qy)W_2\,dp\,dq \quad .... (10)$$

with,

$$\left.\begin{aligned}W_1 &= \left[k_2\left\{\frac{a_{12}pq^2}{\alpha} - \frac{p^2q}{a_{66}}\right\} + k_1\left\{\frac{p^3}{a_{66}} + \frac{q^2p}{\alpha}a_{11}\right\}p\right] / W \\ W_2 &= \left[k_2\left\{\frac{pq^2a_{12}}{\alpha} - \frac{q^3}{a_{66}}\right\} - k_1\left\{\frac{pq^2a_{11}}{\alpha} - \frac{qp^2}{a_{66}}\right\}\right] / W\end{aligned}\right\}$$

$$W = q^4 + 2B_1p^2q^2 + B_2p^4, \; B_2 = \frac{a_{22}}{a_{11}}, \; 2B_1 = \frac{(2a_{12} + a_{66})}{a_{11}}$$

T_{cc} is Fourier cosine transform of T w.r.t., x and y, respectively, while T satisfies the equation

$$\left(k_1 \frac{\partial^2}{\partial x^2} + k_2 \frac{\partial^2}{\partial y^2}\right) T = \alpha Q(x, y) \qquad \text{.... (11)}$$

The heat distribution is such that the component of shear stress $\sigma_{xy}^{(h)}$ and $u_y^{(h)}$ are zero at $y = 0$, *i.e.*,

$$u_y^{(h)}(x, 0) = 0, \quad \sigma_{xy}^{(h)}(x, 0) = 0 \qquad \text{.... (12)}$$

Then we shall take Fourier transform of above equation and then substitute in the expressions of $u_x^{(h)}$, $u_y^{(h)}$ above and then using stress-strain relations to get stress component as,

$$\sigma_{yy}^{(h)}(x, y) =$$

$$-\frac{4}{\pi^2} \int_0^\infty \int_0^\infty q^2 Q_{cc} \cos px \cos qy \frac{\left[k_2 (k_3 p^2 - q^2 k_4) + k_1 (p^2 k_5 - q^2 k_6)\right]}{W} dp\, dy \qquad \text{....(13)}$$

where,

$$k_3 = \alpha a_{22} - a_{66}, \quad k_4 = \alpha a_{11}, \quad k_6 = \alpha a_{12}, \qquad k_5 = a_{11}(\alpha - 2a_{11}a_{66})$$

3. Elasticity Problem

We shall follow the method of Kushwaha [9] to obtain the solution of elasticity problem.

We take displacement components as,

$$u_x^{(e)}(x, y) = \frac{2}{\pi} \int_0^\infty p^{-1} \left[a_{11} H_{,yy} - p^2 a_{12} H\right] \sin px\, dp \qquad \text{.... (14)}$$

$$u_y^{(e)}(x, y) = \frac{2}{\pi} \int_0^\infty p^{-2} \left[a_{11} H_{,yyy} - p^2 (a_{12} + a_{66}) H_{,y}\right] \cos px\, dp \qquad \text{.... (15)}$$

with (,) is differentiation w.r.t. y and H is given as

$$(\gamma_1 - \gamma_1) H(p, y) = \{(\gamma_1 - \gamma_2) A(p) - B(p)\} e^{-p\gamma_1 y} + B(p) e^{-\gamma_2 p y}, \qquad \text{.... (16)}$$

with A(p) and B(p) are two arbitrary constants and γ_1^2, γ_2^2 are two roots of equations,

$$\gamma^4 - 2B_1\gamma^2 + B_2 = 0$$

Now making use of stress-strain relations (8) for $\sigma_{xy}^{(e)}$ and $\sigma_{yy}^{(e)}$ with (14) – (16), we get as,

$$\sigma_{xy}^{(e)}(x, y) = \frac{2}{\pi(\gamma_1 - \gamma_2)} \int_0^\infty p \sin px \Big[\gamma_1 \left\{ (\gamma_1 - \gamma_2) A(p) - B(p) \right\} e^{-\gamma_1 py}$$

$$+ B(p) e^{-\gamma_2 py} \gamma_2 \Big] dp \qquad .... (17)$$

$$\sigma_{yy}^{(e)}(x, y) = \frac{2}{\pi\alpha} \int_0^\infty p^2 \cos px \Big[\alpha_1 \left\langle \left\{ (\gamma_1 - \gamma_2) A(p) - B(p) \right\} e^{-r_1 py} \right\rangle$$

$$+ B(p) e^{-r_2 py} \Big\rangle - \alpha_2 \left\{ \left\langle (\gamma_1 - \gamma_2) A(p) - B(p) \right\rangle e^{-r_1 py} + B(p) e^{-r_2 py} \right\} dp$$

$$.... (18)$$

$$\alpha_1 = \frac{(a_{11} - a_{12})}{\gamma_1 - \gamma_2}, \quad \alpha_2 = \frac{a_{12}}{r_1 - \gamma_2}$$

Now, after making use of (12) in boundary conditions (5) - (6), they are reduced to

$$\sigma_{xy}^{(e)}(x, 0) = 0, 0 \le x < \infty \qquad .... (19)$$

$$u_{xy}^{(e)}(x, 0) = 0, x \in I_2^+ \cup I_4^+ \qquad .... (20)$$

$$\sigma_{yy}^{(e)}(x, 0) = -\sigma_{yy}^{(h)}(x, 0), \ x \in I_1^+ \cup I_3^+ \qquad .... (21)$$

4. Reduction to and solution of quintuple integral equation

Reduction

The boundary condition (19) along with (17) gives

$$\gamma_1 A(p) = B(p) \qquad ....(21)$$

The boundary condition (20) and relation (15)-(16) and then using (21) we get,

$$\alpha_3 \int_0^\infty p\, B(p) \cos px\, dp = 0, x \in I_2^+ + I_4^+, \quad \text{.... (22)}$$

The condition (21) and relation (18)give,

$$\int_0^\infty p^2 \cos(px)\, B(p) dp = -\frac{\pi\alpha}{2\alpha_4} \sigma_{yy}^{(h)}(x,0), x \in I_1^+ + I_3^+ \quad \text{.... (23)}$$

with,

$$\alpha_4 = \frac{\left(\gamma_1^2\alpha_1 - \alpha_2\gamma_2^2\right)}{\gamma_1} + \left(\gamma_2^2 - \gamma_1^2\right)\alpha_1$$

$$\alpha_3 = \frac{2}{\pi}\left[(\gamma_1 + \gamma_2)^2 - \gamma_1\gamma_2 + 2(a_{12} + a_{66})\right]$$

Thus, the physical problem is reduced to the solution of quintuple integral equations (22) – (23).We assume

$$p\, B(p) = \phi(p)$$

Then using above in (22) – (23) we get

$$\int_0^\infty \phi(p) \cos px\, dp = 0, x \in I_2^+ + I_4^+ \quad \text{.... (24)}$$

and

$$\int_0^\infty p\, \phi(p) \cos px\, dp = -\alpha_5 \sigma_{yy}^{(h)}(x,0), \quad \alpha_5 = \frac{(\pi\alpha)}{(2\alpha_4)} \quad \text{.... (25)}$$

Solution

We take trial solution of (24) – (25) as, see Kushwaha [10],

$$\pi\phi(p) = 2\left[\left\langle \int_0^b g_1(t) + \int_c^d g_2(t) \right\rangle \frac{\sin pt\, dt}{p}\right. \quad \text{.... (26)}$$

Now we use (26) in (24), then use the integral

$$\int_0^\infty \frac{\sin px \cos qx}{x} dx = \begin{cases} \frac{\pi}{2}, p > q \\ \frac{\pi}{4}, p = q \\ 0, p < q \end{cases}$$

and taking $g_1(0) = 0,$ with no loss of generality, the equation (24) is satisfied identically if,

$$\int_c^d g_2(t)dt = 0 \quad \text{.... (27)}$$

The substitution of (26) in (25) and using,

$$\int_0^\infty \frac{\sin px \sin qx}{x} dx = \frac{1}{2}\log\left|\frac{p+q}{p-q}\right|$$

We get

$$g_1(t) = \frac{2t}{\pi^2 \psi(t)}[\Delta(t)], t \in I_1^+ \quad \text{.... (28)}$$

$$g_2(t) = -\frac{2t}{\pi^2 \psi(t)}\Delta(t), t \in I_3^+ \quad \text{.... (29)}$$

$$\Delta(t) = \left\langle \int_d^b - \int_c^d \right\rangle \frac{p(x)\,\psi(x)}{x^2 - t^2} dx + D_0, \quad (30)$$

$$p(x) = \alpha_5 \sigma_{yy}^{(h)}(x, 0) \quad \text{.... (30)a}$$

where D_0 is an arbitrary constant to be evaluated through (27). And

$$\psi(t) = \left\{\left|\left(b^2 - t^2\right)\left(t^2 - c^2\right)\left(d^2 - t^2\right)\right|\right\}^{\frac{1}{2}}, \quad \text{.... (31)}$$

5. Physical Quantities

The quantities of physical importance are the components of stress and of displacement in the vicinity of crack tips.

Crack Shape

The crack opening displacement is evaluated through the left hand side of (22) for $x \in I_1^+ + I_3^+$. We evaluate the integral with the help of (26) and is given as,

$$u_y^{(e)}(x,0) = \frac{\alpha_3 \pi}{2} \begin{cases} \int_x^b g_1(t)dt, x \in I_1^+ \\ -\int_x^d g_2(t)dt, x \in I_3^+ \end{cases} \quad \text{.... (32)}$$

Stress Component

Normal stress component is given through

$$\sigma_{yy}(x,0) = \sigma_{yy}^{(h)}(x,0) + \sigma_{yy}^{(e)}(x,0),\ x \in I_2^+ + I_4^+$$

$\sigma_{yy}^{(h)}(x,0)$ will be evaluated through (13) and is given as

$$\sigma_{yy}^{(e)}(x,0) = \frac{2}{\pi}\frac{\alpha_4}{\alpha}\int_0^\infty p\phi(p)\cos px\, dp,\ x \in I_2^+ + I_4^+$$

Now, using (26) in above and evaluating integral we get

$$\sigma_{yy}^{(e)}(x,0) = \frac{2}{\pi} \cdot \frac{\alpha_4}{\alpha} \begin{cases} \dfrac{x\Delta(x)}{\psi(x)}, x \in I_2^+ \\ -\dfrac{x\Delta(x)}{\psi(x)}, x \in I_4^+ \end{cases} \quad \text{.... (33)}$$

Where $\Delta(x)$ and $\psi(x)$ are defined in (30) –(31),respectively.

Stress-Intensity Factors

The stress-intensity factors are very important in fracture design parameter. These are defined at crack tips as,

$$K_b = \lim_{x \to b^-} \sqrt{x-b}\ \sigma_{yy}(x,0),\quad K_c = \lim_{x \to c^+} \sqrt{c-x}\,\sigma_{yy}(x,0)$$

Thus,

$$\left.\begin{aligned} K_b &= \lim_{x\to b^-} \sqrt{x-b}\ \sigma_{yy}^{(h)}(x,0) + \frac{1}{\pi}\frac{\alpha_4}{\alpha}\frac{\sqrt{2b}}{\psi_1(b)}\Delta_1(b) \\ K_c &= \lim_{x\to c^+} \sqrt{c-x}\ \sigma_{yy}^{(h)}(x,0) + \frac{\alpha_4}{\pi\alpha}\frac{\sqrt{2c}}{\psi_2(c)}\Delta_1(c) \\ K_d &= \lim_{x\to d^-} \sqrt{x-d}\ \sigma_{yy}^{(h)}(x,0) + \frac{\alpha_4}{\pi\alpha}\frac{\sqrt{2b}}{\psi_3(c)}\Delta_1(b) \end{aligned}\right\} \quad \text{.... (34)}$$

with

$$\psi_1(b) = \left[\left(c^2-b^2\right)\left(d^2-b^2\right)\right],\ \psi_2(c) = \left[\left(c^2-b^2\right)\left(d^2-c^2\right)\right]^{\frac{1}{2}}$$

$$\psi_3(d) = \left[\left(d^2-c^2\right)\left(d^2-b^2\right)\right]^{\frac{1}{2}}$$

The heat distribution is due to heat source therefore, $\sigma_{yy}^{(h)}(x,0)$, is not singular at crack tips.

6. Special case of point Heat source

Since the rivets and stiffeners are simulated by point forces in the medium, therefore, heated rivets/stiffeners can be simulated by point heat source. Point heat source is defined as, see figure-2,

$$Q(x,y) = Q_0\,\frac{\delta(x)}{2}\{\delta(y-h)+\delta(y+h)\} \quad \text{.... (35)}$$

which means that the point heat sources of strength Q_0 and is acting at points $(0,\pm h)$. Taking Fourier cosine transform of above with respect to x and y, we get

$$Q_{cc}(p,q) = Q_0\cos(qh)$$

Substituting the value of Q_{cc}, in (13) and evaluating the integrals,

$$\sigma_{yy}^{(h)}(x,0) = \frac{Q_0 h}{\pi\alpha a_{66}}\left[\frac{C+\alpha k_2}{h^2+k_2^2x^2} + \frac{D}{h^2+\gamma_1 x^2} + \frac{G}{h^2+\gamma_2 x^2}\right] \quad \text{.... (36)}$$

where C, D and G are to be evaluated through, C + D = 1,

$$C\gamma_1\gamma_4 + Dk_2\gamma_2 + Gk_3\gamma_1 = \gamma_3 + \gamma_4$$

$$\gamma_1 = B_1 + \sqrt{B_1^2 - B_2}\,, \gamma_2 = B_1 - \sqrt{B_1^2 - B_2}\,,$$

$$\gamma_3 = D_1 + \sqrt{D_1^2 - D_2}\,, \gamma_4 = D_1 - \sqrt{D_1^2 - D_2}\,,$$

$$D_1 = k_7 k_8\,, D_2 = k_7 k_9\,, k_7 = k_1 a_{12}\,, \alpha$$

$$k_8 = k_2 k_3 + k_1 k_5\,, k_9 = k_2 a_{11}\alpha$$

Thus we see that $\sigma_{yy}^{(h)}(x,0)$ is not singular.

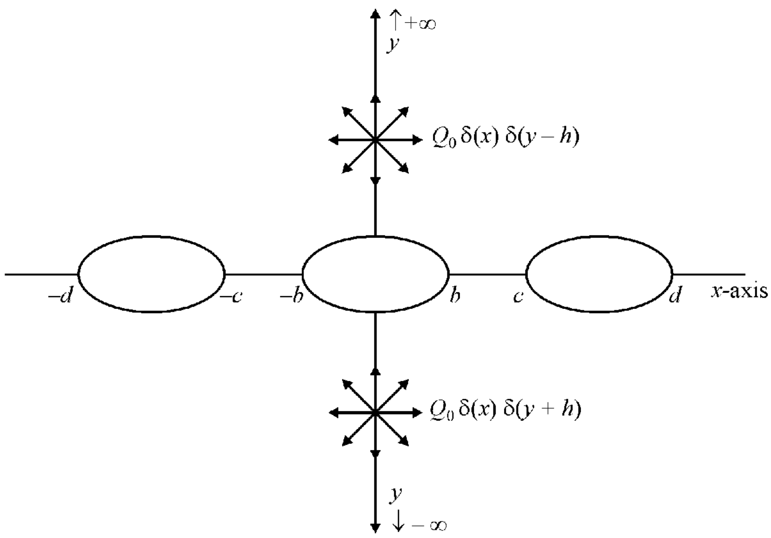

Figure 2. $Q(x, y) = Q_0\,\delta(x)\,\{\delta(y-h) + \delta(y+h)\}$. **The Heat source is of strength** Q_0 **acting at** $(0, \pm h)$.

Stress-Component

$\sigma_{yy}^{(e)}(x,0)$, $x \in I_2^+ + I_4^+$ is to be evaluated through (33) and $\Delta(x)$ from (30) for $x \in I_2^+ + I_4^+$, which is given as below

$$\Delta(x) = \alpha_5 \frac{Q_0 h\, x}{\pi\alpha a_{66}}\left[\delta_4(h_1, x)B_0 + D_0\delta_4(h_2, x) + F_0\delta_4(h_3, x)\right] + M_0\,, \quad (37)$$

$$\delta_4(y,x) = \frac{\left[(c^2+y^2)E\left(\frac{\pi}{2},m_0\right) + \langle\psi_1(x)+\psi_{11}(y)\rangle F\left(\frac{\pi}{2},m_0\right) + \psi_{12}(y)\right]}{(y^2+x^2)}$$

$$m_0^2 = \frac{c^2-b^2}{a^2-b^2},\ \psi_1(x) = \left[\left|(c^2-x^2)(d^2-x^2)\right|\right]^{\frac{1}{2}}$$

$$\psi_{12}(y) = \left[\left|(y^2-b^2)(y^2-c^2)(y^2-d^2)\right|\right]^{\frac{1}{2}}$$

$$\psi_{11}(y) = \left[(b^2+y^2)(d^2+y^2)\right]^{\frac{1}{2}},\ h_1 = \frac{h}{\sqrt{k_1k_2}}, h_2 = \frac{h}{\sqrt{\gamma_1}}, h_3 = \frac{h}{\sqrt{\gamma_2}}$$

$$M_0 = \frac{\left[B_0\Pi_1(h_1) + D_0\Pi_1(h_2) + F_0\Pi_1(h_3)\right]}{F_2}$$

$$F_2 = F\left(\frac{\pi}{2},\mu_3\right), \Pi_1(y) = \frac{\psi_{12}(y)}{y^2+c^2}\Pi\left(\frac{\pi}{2}, m_2 y, m_3\right),\ m_3^2 = \frac{d^2-c^2}{c^2-b^2}$$

$$m_2^2 y^2 = \frac{a^2-c^2}{y^2+c^2},\ B_0 = \frac{bk_2}{\sqrt{k_1k_2}}, D_0 = \frac{D}{\sqrt{r}}, F_0 = \frac{G}{\sqrt{r}}$$

where F, E and Π are complete elliptic integrals, see gradstyen and Ryzhik [6], of first, second and third types respectively.

Thus knowing $\Delta(x)$, $\sigma_{yy}^{(e)}(x,0)$, for $x \in I_2^+ + I_4^+$ is evaluated through (33) and then used the definition of stress-intensity factors which are given in (34)

$$\left.\begin{aligned} K_b &= K_b^{(e)} = \frac{\alpha_4}{\pi\alpha}\frac{\sqrt{2b}}{\psi_1(b)}\Delta_1(b), K_c = K_c^{(e)} = \frac{\alpha_4}{\pi\alpha}\frac{\sqrt{2c}}{\psi_2(c)}\Delta_1(c) \\ K_d &= K_d^{(e)} = \frac{\alpha_4}{\pi\alpha}\frac{\sqrt{2d}}{\psi_3(d)}\Delta_1(d), \Delta_1(x) = \frac{\Delta(x)}{x}, \end{aligned}\right\} \quad ...(38)$$

Crack Opening Displacement

The crack opening displace $u_y^{(e)}(x,0)$ is given through (32), where $g_1(t)$ and $g_2(t)$ are given in (28) – (29) and $\Delta(t)$ is evaluated through (30, (30)a and (36)

$$\Delta(t) = \frac{Q_0 h\alpha_5}{\pi\alpha a_{66}}\left[\sum_{i=1}^{3}\frac{E_i}{h_i^2+t^2}\left\{a_2(h_i^2+t^2)+a_3\left\langle\left|(b^2-t^2)(c^2-t^2)\right|-(b^2+h_i^2)(c^2+h_i^2)\right\rangle\right.\right.$$

$$\left.+a_4\prod_{i=1}^{3}(b^2+h_i^2)\right]+M_{0,}t\in I_1^+ + I_3^+ \qquad \text{.... (39)}$$

Thus displacement will be evaluated through numerical method for integration. And,

$$a_2 = -\pi\left[1+\frac{\sqrt{\alpha^2+\alpha(2d^2-b^2)+1}}{2\alpha\sqrt{(2c^2-b^2)^2-4}}\right]$$

$$\alpha = (2c^2-b^2)+\sqrt{(2c^2-b^2)-4}$$

$$a_3 = -\frac{\pi}{2}+\sqrt{d^2-b^2}F\left(\frac{\pi}{2},m_1\right),m_1^2 = \frac{d^2-c^2}{d^2-b^2}$$

$$a_4 = 2(a^2-b^2)F\left(\frac{\pi}{2},m_1\right)-E\left(\frac{\pi}{2},m_1\right)-\frac{1}{(a^2-b^2)^{3/2}}\Pi\left(\frac{\pi}{2},m_1,m_1\right)$$

Where F, E and Π are complete elliptic integrals of first, second and third type.

7. Discussion and Conclusion

Discussion

Heat flow is through conduction andis governed by equation (11). Heat flow is because of heat source/sink, $Q(x, y)$. If there is no heat source/sink, then equation (11) will reduce to second order homogeneous partial different equation. Then we are to solve by another method. The boundary conditions (12) over heat introduces the symmetry. If these conditions are removed, then problem becomes difficult but not impossible.

Conclusion

Present method can be extended to the problem of three Griffith-cracks in an orthotropic strip. This problem will be reduced to quadruple series equations. Whose solution can be obtained by the method of Kushwaha [10]. There will be a closed form solution and will be reported in next research paper.

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