Journal of Rajasthan Academy of Physical Sciences
ISSN : 0972-6306; URL : http://raops.org.in
Vol. 12, No. 4, Dec. 2013, pp 373-384

A SINGLE SERVER QUEUEING MODEL WITH FEEDBACK AND BALKING

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Received : Oct. 1, 2013

Abstract : This paper presents an analysis for a single server queueing model with feedback and balking in which the inter-arrival time and service time follows exponential distribution. The steady-state queue length probabilities are obtained. Laplace transform of the generating function of the transient-state queue length probabilities of the queueing model are obtained. Some special cases of interest are also derived.

Keywords : Feedback, balking, Laplace transformation, generating function, exponential distribution

2010 Mathematics Subject Classification : 90Bxx, 90B20, 60525

1. Introduction

It is seen that in a fast paced World, these day's customers are very hard pressed for time and hence would usually prefer not to be involved in the act of waiting in any form. However queueing and waiting for service is unavoidable in real life. Consequently customers react to waiting for service in different ways. This reaction is a reflection of their impatience. One of the modes in which customer displays his impatience is balking. Balking is referred to reluctance of a customer to join the queue. A situation of balking is shown in conditions like impatient telephone switchboard customers and hospital emergency room handling critical patients. In this paper, we are going to generalize queueing model with combine effect of feedback and balking. Feedback in queueing literature represents customer's dissatisfaction because of inappropriate quality of service. In case of feedback after getting partial or incomplete service, customer retries for service. The served unit either leaves the system or rejoins it with definite probability. Finch[3] introduced the concept of feedback through his paper "Cyclic queues with feedback". By balking we mean the phenomenon of a customer arriving for service into a non-empty queue and leaving without joining the queue, when he finds too many customers lined up in the system. The factors which influence the decision of a person to join a queue or not may be considered under two general headings (a) those relating to importance of the work for which the customer has arrived (b) those relating to the obstacle which the queue presents, namely the waiting time which he must experience.

The combined effects of balking and reneging in the M/M/1/N queues have been investigated by Ancker and Gafarian [1],[2]. Wang and Chang [9] generalized this work to study M/M/C/N queues. Depending upon generating function technique, Kumar and Parthasarathy [7] investigated transient solution of M/M/1 queue with balking. Tarabia [8] analyzed transient and steady state solution of an M/M/1 queue with balking, catastrophes, server failure and repairs.

Here we have generalized the work of Garg and Singla [4] for single server by applying the concept of balking to take into consideration the broader perspective of customer's impatience. We are going to consider single server queueing model with feedback and balking with infinite waiting capacity. In this paper, we consider $M/M/1/\infty$ queueing model with feedback and balking and obtained steady-state queue length probabilities for the problem. Laplace transform of the probability generating function of the transient-state queue length probabilities have also been obtained.

The practical situation which corresponds to the above model can be that of an arriving customer to a chemist shop for buying medicines. On seeing long queue at the shop an arriving customer may leave the system that is balking. The shopkeeper satisfies the customer either in one service or in two services. The customers after getting the first service, if found unsatisfactory (for example if he finds the given medicine is expired one or he forgot to buy some other medicine) is again sent in the queue for service. The shopkeeper can know the various probabilities for the number of customers to be served at any time.

The queueing system studied in this paper is governed by the following assumptions

1. Arrivals are Poisson with parameter λ and the service time distribution of every unit is Exponential with parameter μ
2. The probability of rejoining the system is p and that of leaving the system is q for the units getting first service, so that $p + q = 1$. However the unit will have to leave the system after getting second service.
3. The probability that the unit joins the service channel for the first time is assumed to be c_1 and that for the second time is c_2, so that $c_1 + c_2 = 1$.
4. On arrival, a customer either decides to join the queue with probability. The arriving customer joins the queue definitely, if the number of customers in the system is less than 'k'. It may balk with probability 1-β or joins the queue with probability (β) if the number of customers in the system size is $\geq k$.
5. The waiting space is infinite.
6. The stochastic processes involved, viz.
   a) arrivals of units
   b) departure of units , are statistically independent

2. Definitions

$P_n^{(0)}(t) =$ Probability that there are n units in the system at any time $t$and the next unit is to depart for the first time.

$P_n^{(1)}(t) =$ Probability that there are n units in the system at any time t and the next unit is to depart for second time.

$P_n(t) =$ Probability that there are n units in the system at any time t.

$$P_n(t) = P_n^{(0)}(t) + P_n^{(1)}(t), \quad n \geq 0 \qquad .....(1)$$

Initially,

$$P_0^{(0)}(0) = 1 \text{ and } P_0^{(1)}(t) = 0, \qquad n \geq 0$$

The difference differential equations describing the system are

$$\frac{d}{dt} P_0^{(0)} = -\lambda P_0^{(0)}(t) + \mu q P_1^{(0)}(t) + \mu P_1^{(1)}(t), \; n = 0 \qquad ....(2)$$

$$\frac{d}{dt} P_n^{(0)}(t) = -(\lambda + \mu) P_n^{(0)}(t) + \lambda P_{n-1}^{(0)}(t) + \mu c_1 q P_{n+1}^{(0)}(t) + \mu c_1 P_{n+1}^{(1)}(t)$$

$$+ \mu c_1 p(1 - \delta_{n,1}) P_n^{(0)}(t), \qquad 0 < n \leq k-1 \qquad ......(3)$$

$$\frac{d}{dt} P_n^{(1)}(t) = -(\lambda + \mu) P_n^{(1)}(t) + \lambda P_{n-1}^{(1)}(t) + \mu c_2 q P_{n+1}^{(0)}(t) + \mu c_2 P_{n+1}^{(1)}(t)$$

$$+ \mu(c_1 \delta_{n,1} + c_2) p P_n^{(0)}(t), \qquad 0 < n \leq k-1 \qquad ....(4)$$

$$\frac{d}{dt} P_n^{(0)}(t) = -(\lambda\beta + \mu) P_n^{(0)}(t) + [\lambda\beta + \lambda\delta_{n-k+1,1}(1-\beta)] P_{n-1}^{(0)}(t) + \mu c_1 q P_{n+1}^{(0)}(t)$$

$$+ \mu c_1 P_{n+1}^{(1)}(t) + \mu c_1 p P_n^{(0)}(t), \qquad n \geq k \qquad ....(5)$$

$$\frac{d}{dt} P_n^{(1)}(t) = -(\lambda\beta + \mu) P_n^{(1)}(t) + [\lambda\beta + \lambda\delta_{n-k+1,1}(1-\beta)] P_{n-1}^{(1)}(t) + \mu c_2 q P_{n+1}^{(0)}(t)$$

$$+ \mu c_2 P_{n+1}^{(1)}(t) + \mu c_2 p P_n^{(0)}(t), \qquad n \geq k \qquad ....(6)$$

where $\delta_{n,1} = \begin{cases} 1, & \text{for } n = 1 \\ 0, & \text{otherwise} \end{cases}$

The steady-state difference equations describing the system are

$$\rho P_0^{(0)} = q P_1^{(0)} + P_1^{(1)} \qquad ....(7)$$

$$(\rho + 1) P_n^{(0)} = \rho P_{n-1}^{(0)} + c_1 q P_{n+1}^{(0)} + c_2 P_{n+1}^{(1)} + c_1(1 - \delta_{n,1}) p P_n^{(0)}, \qquad 0 < n \leq k-1 \qquad ....(8)$$

$$(\rho+1)P_n^{(1)}=\rho P_{n-1}^{(1)}+c_2qP_{n+1}^{(0)}+c_2P_{n+1}^{(1)}+(c_1\delta_{n,1}+c_2)pP_n^{(0)}, \quad 0<n\le k-1 \quad ....(9)$$

$$(\rho\beta+1)P_n^{(0)}=[\rho\beta+\rho\delta_{n-k+1,1}(1-\beta)]P_{n-1}^{(0)}+c_1qP_{n+1}^{(0)}+c_1P_{n+1}^{(1)}+c_1pP_n^{(0)},\ n\ge k \ ....(10)$$

$$(\rho\beta+1)P_n^{(1)}=[\rho\beta+\rho\delta_{n-k+1,1}(1-\beta)]P_{n-1}^{(1)}+c_2qP_{n+1}^{(0)}+c_2P_{n+1}^{(1)}+c_2pP_n^{(0)},\ n\ge k \ ....(11)$$

Taking the Laplace Transformation $\overline{P}_n(S)=\int_0^\infty e^{-st}P_n(t)dt;\ \text{Re}\,s>0$ of (2) - (6) and dividing by μ

$$\left(\rho+\frac{s}{\mu}\right)\overline{P}_0^{(0)}(s)=\frac{1}{\mu}+q\overline{P}_1^{(0)}(s)+\overline{P}_1^{(1)}(s) \quad ....(12)$$

$$\left(\rho+\frac{s}{\mu}+1\right)\overline{P}_n^{(0)}(s)=\rho\overline{P}_{n-1}^{(0)}(s)+c_1q\overline{P}_{n+1}^{(0)}(s)+c_1\overline{P}_{n+1}^{(1)}(s)+c_1p(1-\delta_{n,1})\overline{P}_n^{(0)}(s)$$

$$0<n\le k-1 \quad .....(13)$$

$$\left(\rho+\frac{s}{\mu}+1\right)\overline{P}_n^{(1)}(s)=\rho\overline{P}_{n-1}^{(1)}(s)+c_2q\overline{P}_{n+1}^{(0)}(s)+c_2\overline{P}_{n+1}^{(1)}(s)+(c_1\delta_{n,1}+c_2)p\overline{P}_n^{(0)}(s)$$

$$0<n\le k-1 \quad .....(14)$$

$$\left(\rho\beta+\frac{s}{\mu}+1\right)\overline{P}_n^{(0)}(s)=[\rho\beta+\rho\delta_{n-k+1,1}(1-\beta)]\overline{P}_{n-1}^{(0)}(s)+c_1q\overline{P}_{n+1}^{(0)}(s)+c_1\overline{P}_{n+1}^{(1)}+c_1\overline{P}_n^{(0)}(s)$$

$$n\ge k \quad .....(15)$$

$$\left(\rho\beta+\frac{s}{\mu}+1\right)\overline{P}_n^{(1)}(s)=[\rho\beta+\rho\delta_{n-k+1,1}(1-\beta)]\overline{P}_{n-1}^{(1)}(s)+c_2q\overline{P}_{n+1}^{(0)}(s)+c_2\overline{P}_{n+1}^{(1)}(s)+c_2p\overline{P}_n^{(0)}(s)$$

$$n\ge k \quad .....(16)$$

Definitions

$P^0(z,t)=\sum_{n=0}^{\infty}P_n^{(0)}(t)z^n$; $\qquad P^1(z,t)=\sum_{n=0}^{\infty}P_n^{(1)}(t)z^n$

$P(z,t)=P^0(z,t)+P^{(1)}(z,t)$; $\qquad \overline{P}^0(z,s)=\int_0^\infty e^{-st}P^0(z,t)dt$

$\overline{P}^1(z,s)=\int_0^\infty e^{-st}P^1(z,t)dt$; $\qquad \overline{P}(z,s)=\int_0^\infty e^{-st}\overline{P}(z,t)dt$ with $|z|\le 1$

3. Steady-state solution of the problem

Using $Ef(x)=f(x+1)$, equation (10) and (11) becomes

$$[c_1q(E)^2+\{c_1p-(\rho\beta+1)\}E+\rho\beta]P_n^{(0)}+c_1E^2P_n^{(1)}=0, \qquad n\geq k+1 \qquad .....(17)$$

$$[c_2q(E)^2+c_2pE]P_n^{(0)}+[c_2E^2-(\rho\beta+1)E+\rho\beta]P_n^{(1)}=0, \qquad n\geq k+1 \qquad .....(18)$$

To have solution of above system of equations, we must have:

$$(E-1)\{(\rho\beta+1)(c_1q+c_2)E^2-\rho\beta(\rho\beta+2-c_1p)E+(\rho\beta)^2\}=0, \; n\geq k+1 .....(19)$$

The values of $P_n^{(0)}$ and $P_n^{(1)}$ are given by

$$P_n^{(0)}=\sum_{i=0}^{2}a_iz^n \text{ and } P_n^{(1)}=\sum_{i=0}^{2}b_iz^n \text{ for } n\geq k+1$$

where z_0, z_1, z_2 are the roots of (19) with $z_0=1$ and $a_i, b_i; i=0,1,2$ are arbitrary constants to be evaluated. The other two root of equation (19) are

$$z_1=\frac{\rho\beta}{\rho\beta+1} \text{ and } z_2=\frac{\rho\beta}{c_2+c_1q}, \qquad \rho=\frac{\lambda}{\mu}<1 \qquad ....(20)$$

z_1 is always less than 1 but z_2 is less than 1 only when $\rho\beta<(c_2+c_1q)$. So we have two or one root <1 accordingly if $\rho\beta<(c_2+c_1q)$ or not. In case $z_2\geq 1$ take $a_2=b_2=0$.

To have convergence of the solution a root ≥ 1 must be rejected.

Thus rejecting , we have

$$P_n^{(0)}=\sum_{i=0}^{2}a_iz^n \text{ and } P_n^{(1)}=\sum_{i=0}^{2}b_iz^n \text{ for } n\geq k+1$$

From (10) and (11) for n = k +1 we can get probabilities $P_k^{(0)}$ and $P_k^{(1)}$. Subsequently putting the values in equation (10) and (11) for $n=k$ we can get $P_{k-1}^{(0)}$ and $P_{k-1}^{(1)}$. Further substituting these values in equations (8) and (9), we get the probabilities $P_{k-2}^{(0)}, P_{k-2}^{(1)}, P_{k-3}^{(0)}, P_{k-3}^{(1)}, ...P_1^{(0)}$ and $P_1^{(1)}$ in terms of $P_0^{(0)}$. Four unknowns a_1, a_2, b_1 and b_2 (two unknown a_1 and b_1 in case of $z_2\geq 1$) can be evaluated from equations (7), {(8), (9) for n=1} and {(10) for $n = k + 2$} in terms of $P_0^{(0)}$ and the value of $P_0^{(0)}$ can be found using the relation $P_0^{(0)} = 1 - \sum_{n=1}^{\infty}(P_n^{(0)} + P_n^{(1)})$.

Hence by using the value of a_1, a_2, b_1 and b_2 and $P_0^{(0)}$ the probabilities $P_n^{(0)}$ and $P_n^{(1)}$ are completely known for various values of n.

4. Special Cases

(i) When there is no feedback : Putting $q = 1,\ p = 0, c_1 = 1, c_2 = 0, P_n^{(1)} = 0,$ $P_n^{(0)} = P_n$ then equation (7)–(11) becomes

$$\rho P_0 = P_1 \qquad \text{.... (21)}$$

$$(\rho + 1)P_n = \rho P_{n-1} + P_{n+1}, \qquad 0 < n \le k - 1 \qquad \text{....(22)}$$

$$(\rho\beta + 1)P_n = [\rho\beta + \rho\delta_{n-k+1,1}(1-\beta)]P_{n-1} + P_{n+1} \qquad n \ge k \qquad \text{....(23)}$$

Using $Ef(x) = f(x+1)$ on equation (23), we get

$$[E^2 - (\rho\beta + 1)E + \rho\beta]P_n = 0 \qquad n \ge k + 1$$

The above auxiliary equation is solved by E =1 and E = $\rho\beta$ i.e. $\frac{\lambda\beta}{\mu} < 1.$

Rejecting E = 1, we get

$$P_n = a(\rho\beta)^n \qquad n \ge k + 1 \qquad \text{....(24)}$$

(where ‘a’ is a constant to be evaluated)

Using $P_{k+1} = a(\rho\beta)^{k+1}$ in equation (23) for n = k, we get

$$P_k = a(\rho\beta)^k$$

and using these in equation (22), we get

$$P_{k-1} = a\rho^{k-1}\beta^k$$

and so on, $P_0 = a\rho^0\beta^k = a\beta^k$

Since $\sum_{i=0}^{n} P_n = 1$ therefore, we have $a = \dfrac{1}{\beta^k \sum_{n=0}^{k-1} \rho^n + \sum_{n=k}^{\infty} (\rho\beta)^n}$

Put in equation (24), we get

$$P_n = \frac{(\rho\beta)^n}{\beta^k \sum_{n=0}^{k-1} \rho^n + \sum_{n=k}^{\infty} (\rho\beta)^n}, \qquad n \geq k+1$$

and
$$P_0 = \frac{\beta^k}{\beta^k \sum_{n=0}^{k-1} \rho^n + \sum_{n=k}^{\infty} (\rho\beta)^n}$$

(ii) When there is no feedback and no balking :

Putting $q = 1, p = 0; c_1 = 1, c_2 = 0; P_n^{(1)} = 0, P_n^{(0)} = P_n, \beta = 1$ in equation (7), then equation (11) becomes

$$\rho P_0 = P_1 \qquad \text{....(25)}$$

$$(\rho + 1)P_n = \rho P_{n-1} + P_{n+1} \qquad \text{...(26)}$$

using $Ef(x) = f(x+1)$ on equation (26), we get

$$[E^2 - (\rho + 1)E + \rho]P_n = 0$$

The above auxiliary equation is solved by E=1 and E=ρ i.e. λ/μ<1.

Therefore rejecting E = 1, we get

$$P_n = a(\rho)^n \qquad n \geq 1$$

Using the value of P_n in equation (25) we get

$$P_0 = a$$

Since $\sum_{n=0}^{\infty} P_n = 1$ hence $a = (1 - \rho)$

Therefore $P_n = (1-\rho)\rho^n$ and $P_0 = (1-\rho)$

which is same as in M/M/1/∞ classical Model.

Laplace transformation of probability generating function of transient - state queue length probabilities

$$\overline{P}^{(0)}(z,s)=\frac{1}{Q(z)}[(1-A)\{(c_1pz-c_2q)z\overline{P}_1^{(0)}(s)-c_2z\overline{P}_1^{(1)}(s)\}-[\{c_1(q+pz)-z\}A$$

$$+c_2z]\overline{P}_0^{(0)}(s)+z/\mu(A-c_2)-\rho z(1-z)(A-c_2)(1-\beta)\sum_{n=0}^{k-1}\overline{P}_n^{(0)}(s)z^n$$

$$-c_1\rho z(1-z)(1-\beta)\sum_{n=0}^{k-1}\overline{P}_n^{(1)}(s)z^n],\quad |z|\le 1 \qquad ....(27)$$

$$\overline{P}_1^{(1)}(z,s)=\frac{1}{Q(z)}[\{zA(c_1pz-c_2q)+c_2qz(q+c_2pz)-c_1pz^2(pz+q)\}\overline{P}_1^{(0)}(s)$$

$$+\{(q+pz)c_2(z-A)\}\overline{P}_0^{(0)}(s)-c_2z[A-(q+pz)]\overline{P}_1^{(1)}(s)+\frac{z}{\mu}\{c_2(q+zp)\}$$

$$-c_2\rho z(1-z)(1-\beta)(q-pz)\sum_{n=0}^{k-1}\overline{P}_n^{(0)}(s)z^n-\rho z(1-z)(1-\beta)[A$$

$$-c_1(q+pz)]\sum_{n=0}^{k-1}\overline{P}_n^{(1)}(s)z^n],\qquad |z|\le 1 \qquad ....(28)$$

$$\overline{P}(z,s)=\frac{1}{Q(z)}[-(1-z)\{pz(c_2q-c_1pz)\overline{P}_1^{(0)}(s)+pzc_2\overline{P}_1^{(1)}(s)+(qA+pzc_2)\overline{P}_0^{(0)}(s)\}$$

$$+(z/\mu)\{A-c_2p(1-z)\}-\rho z\{A+(1-z)c_1p\}(1-z)(1-\beta)\sum_{n=0}^{k-1}\overline{P}_n^{(1)}(s)z^n$$

$$-(1-z)(1-\beta)pz\{A-(1-z)c_2p\}\sum_{n=0}^{k-1}\overline{P}_n^{(0)}(s)z^n],\qquad |z|\le 1 \qquad ....(29)$$

where $Q(z)=\{(A-c_1(q+pz)(A-c_2)\}-(q+pz)c_1c_2,$

$$A=\left\{-\rho\beta z^2+\left(\frac{s}{\mu}+\rho\beta+1\right)z\right\},$$

$$D=K_1(z)K_2(z)-c_1c_2(q+pz),$$

$$K_1(z) = -\rho\beta z^2 + \left(\frac{s}{\mu} + \rho\beta + 1 - c_1 p\right)z - c_1 q$$

and $$K_2(z) = \left(-\rho\beta z^2 + (s/\mu + \rho\beta + 1)z - c_2\right)$$

Obviously $K_1(z)$ and $K_2(z)$ have two zeroes inside the unit circle.

Let $f(z) = K_1(z)K_2(z)$ and $g(z) = (q + pz)c_1 c_2$

Then $$|f(z)| = |(-\rho\beta z^2 + \left(\frac{s}{\mu} + \rho\beta + 1 - c_1 p\right)z - c_1 q)|$$

$$|(-\rho\beta z^2 + \left(\frac{s}{\mu} + \rho\beta + 1\right)z - c_2)|$$

$= (\xi + c_2)(\xi + c_1)$ for $\frac{s}{\mu} = \xi + in, |z| = 1$

$> c_1 c_2 \geq |g(z)|$

Hence $|f(z) > |g(z)|$ on $|z| = 1$

Since all the conditions of Rouche's Theorem are satisfied, so D has two zeroes inside the unit circle. Let these zeroes be $z_m (m = 0,1)$. Numerator must vanish for these two zeroes since $\overline{P}(z,s)$ is an analytical function of z. These two equations along with equation(12) will determine the three unknowns $\overline{P}_0^{(0)}(s), \overline{P}_1^{(0)}(s)$, and $\overline{P}_1^{(1)}(s)$ (in case k =2) along with equations (12) and {(13) and (14) for n =1}will determine the five unknowns $\overline{P}_0^{(0)}(s), \overline{P}_1^{(0)}(s)$, $\overline{P}_1^{(1)}(s), \overline{P}_2^{(0)}(s)$ and $\overline{P}_2^{(1)}(s)$ (in case k=3) and along with the equations (12) and {(13),(14) for k=1,2,3…….k-2} will in general determine the (2k-1) unknowns $\overline{P}_0^{(0)}(s), \overline{P}_1^{(0)}(s)$, $\overline{P}_1^{(1)}(s), \ldots\ldots, \overline{P}_{k-1}^{(0)}(s)\overline{P}_{k-1}^{(1)}(s)$ (when number of customers = k). Hence the generating function $\overline{P}(z,s)$ is completely known.

$\overline{P}_n(s)$ can be obtained by using the following formula

$$\overline{P}_n(s) = \frac{1}{n!}\frac{d^n}{dz^n}\overline{P}(z,s) \quad \text{at } z = 0$$

In either case $P_n(t)$ can be found by inverting the Laplace transform $\overline{P}_n(s)$.

Further $\overline{P}(1,s) = \frac{1}{s}$, as desired and $\overline{P}(0,s) = \lim_{z \to 0} \overline{P}(z,s) = \frac{\text{zero}}{\text{zero}}$

On using L' Hospital's rule, it can be shown that $\overline{P}(0,s) = \overline{P}_0^{(0)}(s)$

SPECIAL CASES

(i) When there is no feedback i.e. M/M/1/∞ model with balking :

Putting $q = 1, p = 0; \overline{P}^{(0)}(z,s) = \overline{P}(z,s), \overline{P}^{(1)}(z,s) = 0$ *and* $\overline{P}_0^0 = \overline{P}_0(s)$, we get

$$\overline{P}(z,s) = \frac{1}{(A-1)}\left[\frac{z}{\mu} - (1-z)\overline{P}_0(s) - (1-z)(1-\beta)\rho z \sum_{n=0}^{k-1} \overline{P}_n(s) z^n\right] \qquad ....(30)$$

$$\left(\rho = \frac{\lambda}{\mu} < 1; |z| \le 1\right)$$

where $A = \{-\rho\beta z^2 + (s/\mu + \rho\beta + 1)z\}$

(ii) When there is no feedback and no balking :

Putting $\beta = 1$ in equation (30), we get

$$\overline{P}(z,s) = \frac{(z/\mu) - (1-z)\overline{P}_0(s)}{A-1} = \frac{(z/\mu) - (1-z)\overline{P}_0(s)}{-\rho z^2 + \left(\rho + \frac{s}{\mu}\right)z + (z-1)}, \qquad ....(31)$$

$\rho = \lambda/\mu < 1; |z| \le 1$

This coincides with equation (28) of Garg and Singla [4]

Acknowledgements

We are highly thankful to Dr. Rakesh Gupta, Professor and Head, Department of Statistics, Chaudhary Charan Singh University, Meerut for his keen interest and encouragement during the preparation of this paper. We are also thankful to Head, Department of Statistics, Punjabi University, Patiala for providing necessary facilities needed for this purpose.

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