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A REVISED FUZZY GOAL PROGRAMMING APPROACH ON MULTI OBJECTIVE LINEAR FRACTIONAL PROGRAMMING PROBLEM

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Abstract In this paper, we present an alternate procedure for solving multi objective linear fractional programming problem (MOLFPP) based on fuzzy goal programming approach with some modifications in the technique suggested by Lachhwani, K. (J. Raj. Acad. Phy. Sc., 12(2), 2013, 139-150). In the proposed revised technique, separate different linear membership functions are defined for numerator and denominator function of each objective functions of MOLFPP. Then achievement of the highest membership value of each of fuzzy goals is formulated by minimizing the sum of the deviational variables. Comparative analysis is also carried out on numerical example to show efficiency of modified approach over earlier method.

Keyword: Multi objective linear fractional programming problem, Fuzzy goal programming, membership function, compromise optimal solution.

2010 Mathematics Subject Classification : 90C29, 90C32

1. Introduction

Multi objective optimization problems concern with programming problems of several conflicting objectives subject to the given set of constraints. Multi objective programming problems (MOPPs) are frequently encountered in large organizations like Government agencies, production plants, transportation companies etc. Numerous methods have been suggested in literature to tackle the multi objective programming problems. Each method appears to have advantages as well as disadvantages.

Recent literature review: In recent years, Jain and Lachhwani [2] obtained the solution of multi objective linear fractional programming problem by converting it into fuzzy programming problem. Pramanik and Roy [6] gave a procedure for solving multi level programming problem in a large hierarchical decentralized organization through linear fuzzy goal programming approach. Ibrahim [1] presented fuzzy goal programming (FGP) algorithm for solving decentralized bi-level multi objective (DBL-MOP) problem with a single decision maker at the upper level and multiple decision makers at the lower level. Lachhwani and Poonia [3] used fuzzy goal programming approach to solve multi level linear fractional programming problem (MLFPP). Recently, Lachhwani [4] suggested fuzzy goal programming approach to solve multi objective quadratic programming problem. Regarding the presently available procedures, a FGP approach seems to be most appropriate for multi objective programming problem.

A multi objective linear fractional programming (MOLFP) problem seeks to optimize more than one objective function in the form of a ratio in which denominator and numerator function both are linear functions. We assume that the set of feasible solutions is a convex polyhedral with a finite number of extreme points and the denominator of the objective functions is non-zero in the constraint set. The aim of this paper is to present a simple and efficient method than earlier method suggested by Lachhwani [2] to solve MOLFPP. The paper is organized as follows: In section 2, we discuss formulation of MOLFPP and related definitions in context of compromise optimal solution. In section 3, we discuss modified FGP approach, Characterization of membership functions and formulate the mathematical models related to it. Comparative analysis based on numerical example is illustrated in next section. Concluding remarks are given in the last section.

2. Problem Formulation

Mathematically, multi objective linear fractional programming (MOLFP) problem can be defined as:

$$\text{Max.}\{Z_1(X), Z_2(X), ..., Z_k(X)\} \qquad ....(1)$$

where $Z_i(X) = \dfrac{(C_i X + \alpha_i)}{(D_i X + \beta_i)} = \dfrac{N_i(X)}{D_i(X)} \qquad \forall\ i = 1,...,k$

subject to, $X \in S = \left\{ X \in R^n \,\middle|\, AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, X \geq 0, b \in R^m, \forall\ i = 1,...,k \right\}$

Here C_i and D_i $(i = 1,...,k)$ are row vectors with n-components, α_i, β_i are scalars, X and b are column vectors with n and m components respectively. It is assumed that $D_i X + \beta_i > 0 \ \ (i = 1,...,k)$) for all $X \in S$.

For our purpose, we here redefine only compromise optimal solution [2] for multi objective programming problem.

Definition 3. For problem (1), a compromise optimal solution is an efficient solution selected by the decision maker (DM) as being the best solution where the selection is based on the DM's explicit or implicit criteria.

Zeleny [7] as well as most authors describes the act of finding a compromise optimal solution to problem as "……an effort or emulate the ideal solution as closely as possible".

Our FGP model for determining compromise optimal (efficient) solution based on the finding of the totality or subset of efficient solutions with the DM, then choosing one best solution on some explicit or implicit algorithm.

3. Proposed revised FGP Methodology

To formulate the fuzzy goal programming models of MOLFPP, the numerator objective $N_i(X)$, $\forall\ i = 1,...,k$ and denominator objective $D_i(X)$, $\forall\ i = 1,...,k$ would be transformed into fuzzy goals by means of assigning an aspiration level to each of them. Then, they are to be characterized by the associated membership functions.

3.1 Characterization of Membership functions

To build membership functions, fuzzy goals and their aspiration levels should be determined first. Using the individual best solution, we find the maximum and minimum values of all the numerator objective functions and denominator objective functions at each best solution.

The maximum values of each $N_i(\overline{X})$ and $D_i(\overline{X})$ $\forall\, i = 1,\ldots,k$ give upper tolerance limit or aspired level of achievement for the membership function of i - th level numerator and denominator objective function respectively. Similarly, the minimum values of each $N_i(\underline{X})$ and $D_i(\underline{X})$ $\forall\, i = 1,\ldots,k$ give lower tolerance limit or lowest acceptable level of achievement for the membership function of i- th level numerator and denominator objective function respectively. Now linear membership functions for the defined fuzzy goals are:

$$\mu_{Z_i}(N_i(\overline{X})) = \begin{cases} 1 & \text{if } \mathrm{N}_i(\overline{X}) \geq \overline{N_i} \\ \dfrac{N_i(\overline{X}) - \underline{N_i}}{\overline{N_i} - \underline{N_i}} & \text{if } \underline{N_i} \leq N_i(\overline{X}) \leq \overline{N_i} \\ 0 & \text{if } \mathrm{N}_i(\overline{X}) \leq \underline{N_i} \end{cases} \quad \forall\, i = 1,\ldots,k \qquad \ldots.(2)$$

$$\mu_{Z_i}(D_i(\overline{X})) = \begin{cases} 0 & \text{if } \mathrm{D}_i(\overline{X}) \geq \overline{D_i} \\ \dfrac{\overline{D_i} - D_i(\overline{X})}{\overline{D_i} - \underline{D_i}} & \text{if } \underline{D_i} \leq D_i(\overline{X}) \leq \overline{D_i} \\ 1 & \text{if } \mathrm{D}_i(\overline{X}) \leq \underline{D_i} \end{cases} \quad \forall\, i = 1,\ldots,k \qquad \ldots.(3)$$

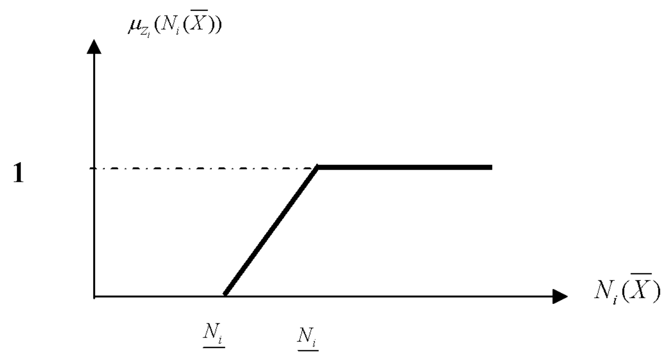


Figure 1(a). Membership functions of maximization type numerator objective functions

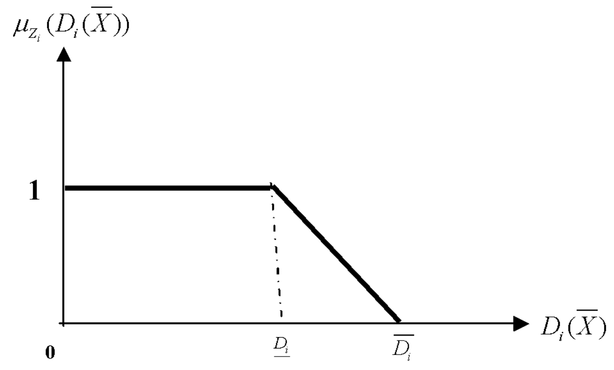


Figure 1(b). Membership functions of maximization type denominator objective functions

Here linear membership functions are considered because these are more suitable than nonlinear ones in context of complex MOLFPPs and consequently, it reduces computational difficulties in proposed method.

3.2 FGP Solution approach

In fuzzy goal programming approaches, the highest degree of membership functions is 1. So, as in Mohamed [5] for the defined membership function in (3), the flexible membership goals with the aspired level 1 can be expressed as:

$$\mu_{z_i}\left(N_i(\mathrm{X})\right) + d_i^{N-} - d_i^{N+} = 1$$

i.e. $$-\overline{N_i} + N_i(\mathrm{X}) + (\overline{N_i} - \underline{N_i})d_i^- - (\overline{N_i} - \underline{N_i})d_i^+ = 0, \forall i = 1,...,k \quad ....(4)$$

$$\mu_{z_i}\left(D_i(\mathrm{X})\right) + d_i^{D-} - d_i^{D+} = 1$$

i.e. $$\underline{D_i} - D_i(\mathrm{X}) + (\overline{D_i} - \underline{D_i})d_i^- - (\overline{D_i} - \underline{D_i})d_i^+ = 0, \forall i = 1,...,k \quad ....(5)$$

where $d_i^{N-}(\geq 0), d_i^{D-}(\geq 0)$ and $d_i^{N+}(\geq 0), d_i^{D+}(\geq 0)$ represent the under and over deviational variables respectively from the aspired levels. It can be easily realized that the membership goals in expression (4)-(5) are inherently linear equation and this may reduce computational difficulties in the solution process. Now, if the most widely used and simplest version of GP (*i.e.* minsum GP) is introduced to formulate the model of the problem (1) under consideration, then FGP model formulation becomes:

Model I : Find X so as to

$$\text{Minimize} \quad \chi = \sum_{i=1}^{k} \left(w_i^N d_i^{N-} + w_i^D d_i^{D-} \right) \qquad (6)$$

Subject to,

$$-\overline{N_i} + N_i(\mathrm{X}) + (\overline{N_i} - \underline{N_i})d_i^- - (\overline{N_i} - \underline{N_i})d_i^+ = 0, \forall\, i = 1,\dots,k$$

$$\underline{D_i} - D_i(\mathrm{X}) + (\overline{D_i} - \underline{D_i})d_i^- - (\overline{D_i} - \underline{D_i})d_i^+ = 0, \forall\, i = 1,\dots,k$$

$$\mathrm{X} \in S = \left\{ X \in R^n \,\middle|\, AX \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, X \ge 0, b \in R^m, \forall\ i = 1,\dots,k \right\}$$

and

$$d_i^{N-}, d_i^{D-}, d_i^{N+}, d_i^{D+} \ge 0, \quad \forall i = 1,\dots,k$$

where χ represents the fuzzy achievement function consisting of the weighted under deviational variables and the numerical weights $w_i^N, w_i^D \ge 0, (\forall i = 1,\dots,k)$ represent the relative importance of achieving the aspired level of the respective fuzzy goals subject to the constraints in the decision making situation. To assess the relative importance of the fuzzy goals properly, the weighted scheme suggested by Mohamed [5] can be used to assign the values to $w_i^N, w_i^D \ge 0, (\forall i = 1,\dots,k)$. In the present formulation $w_i^N, w_i^D \ge 0$ can be determined as:

$$w_i^N = \frac{1}{\overline{N_i} - \underline{N_i}} \qquad \text{....(7)}$$

$$w_i^D = \frac{1}{\overline{D_i} - \underline{D_i}} \qquad \text{....(8)}$$

The above model can also be rewritten as:

Model II Find X so as to

$$\text{Minimize} \quad \chi = \sum_{i=1}^{k} \left(d_i^{N-} + d_i^{D-} \right) \qquad \text{....(9)}$$

Subject to,

$$-\overline{N_i} + N_i(\mathrm{X}) + (\overline{N_i} - \underline{N_i})d_i^- - (\overline{N_i} - \underline{N_i})d_i^+ = 0, \forall\, i = 1,\dots,k$$

$$\underline{D_i} - D_i(\mathrm{X}) + (\overline{D_i} - \underline{D_i})d_i^- - (\overline{D_i} - \underline{D_i})d_i^+ = 0, \forall\, i = 1,\dots,k$$

$$X \in S = \left\{ X \in R^n \;\middle|\; AX \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, X \ge 0, b \in R^m, \forall\, i = 1,\dots,k \right\}$$

and $\quad d_i^{N-}, d_i^{D-}, d_i^{N+}, d_i^{D+} \ge 0, \quad \forall i = 1,\dots,k$

In model II the numerical weights are taken as unity.

Model III Find X so as to

$$\text{Minimize } \chi = \sum_{i=1}^{k} \left(d_i^{N-} + d_i^{D-}\right) \qquad ...(10)$$

Subject to

$$-\overline{N_i} + N_i(\mathrm{X}) + (\overline{N_i} - \underline{N_i})d_i^- \ge 0, \forall i = 1,\dots,k$$

$$\underline{D_i} - D_i(\mathrm{X}) + (\overline{D_i} - \underline{D_i})d_i^- \ge 0, \forall i = 1,\dots,k$$

$$X \in S = \left\{ X \in R^n \;\middle|\; AX \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, X \ge 0, b \in R^m, \forall\, i = 1,\dots,k \right\}$$

and $\quad d_i^{N-}, d_i^{D-}, d_i^{N+}, d_i^{D+} \ge 0, \quad \forall i = 1,\dots,k$

However, model I, II and III can be easily solved using linear programming techniques.

4. Numerical Example

To illustrate the revised approach, we consider the following example as given in [2]:

Example 1. Maximize $\{Z_1(X), Z_2(X)\}$

where $Z_1(X) = \dfrac{(2x_1 + 20x_2 + 12)}{(-2x_1 - 5x_2 + 15)}$

$$Z_2(X) = \frac{(3x_1 + 30x_2 + 51)}{(-4x_1 - 10x_2 + 30)}$$

subject to, $x_1 + 15x_2 \le 2$

$3x_1 + 20x_2 \le 4$ and $x_1, x_2 \ge 0$

Using the proposed revised methodology, the FGP model I is obtained as:

Model I Find X (x_1, x_2) so as to Minimize

$$\chi = 0.3125 d_1^{N-} + 0.3749 d_1^{D-} + 0.1872 d_2^{N-} + 0.1874 d_2^{D-}$$

Subject to,

$$2x_1 + 20x_2 + 3.2 d_1^{N-} - 3.2 d_1^{N+} = 3.2$$

$$2x_1 + 5x_2 + 2.667 d_1^{D-} - 2.667 d_1^{D+} = 2.667$$

$$3x_1 + 30x_2 + 4.8 d_2^{N-} - 4.8 d_2^{D-} = 4.8$$

$$4x_1 + 10x_2 + 5.334 d_2^{D-} - 5.334 d_2^{D-} = 2.334$$

$$x_1 + 15x_2 \le 2$$

$$3x_1 + 20x_2 \le 4$$

and

$$x_1, x_2, d_1^{N-}, d_1^{N+}, d_1^{D-}, d_1^{D+}, d_2^{N-}, d_2^{N+}, d_2^{D-}, d_2^{D+} \ge 0$$

Solving the above problem using linear techniques or software package, the compromise optimal solution obtained as: $x_1 = 1.3333,\ x_2 = 0,\ d_1^{N-} = 0.1666, d_2^{N-} = 0.1666, d_1^{D-} = 0.12498 \times 10^{-3}, d_2^{D-} = 0,\ d_1^{N+} = 0,$ $d_2^{N+} = 0,\ d_1^{D+} = 0,$ $d_2^{D+} = 0.5627$ with values of objective functions $Z_1(\overline{X}) = 1.1692, Z_2(\overline{X}) = 2.1461.$

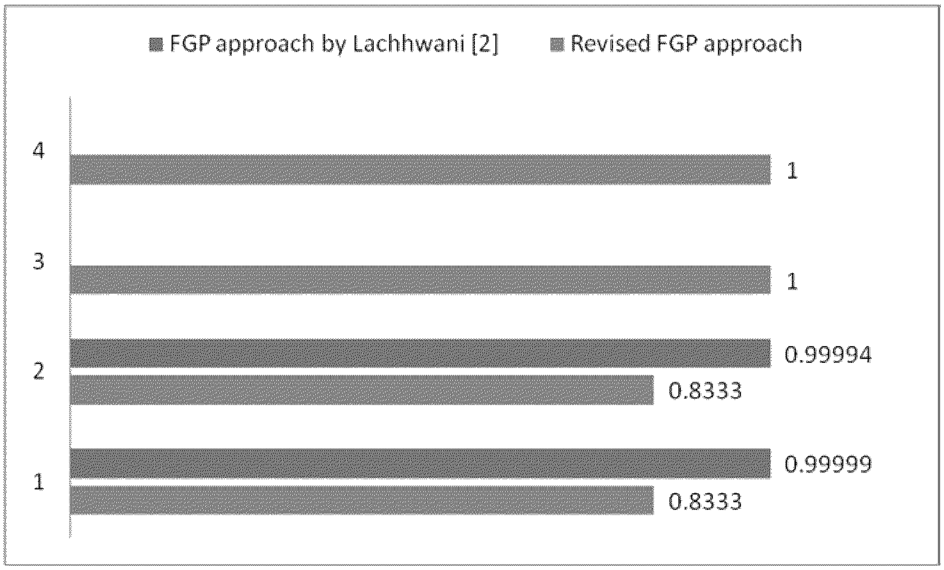
Also achieved values of membership functions are: $\mu_{z_1}(N_1(\text{X})) = 0.8333,$ $\mu_{z_2}(N_2(\text{X})) = 0.8331,$ $\mu_{z_1}(D_1(\text{X})) = 1$ and $\mu_{z_2}(D_2(\text{X})) = 1.$

Note that the compromise optimal solution of the problem using our earlier technique suggested by Lachhwani [2] obtained as: $x_1 = 1.33333,$ $x_2 = 0$ $d_1^- = 0.24170 \times 10^{-5}, d_2^- = 0.45198 \times 10^{-5},$ $d_1^+ = 0,$ $d_2^+ = 0,$ $Z_1(\overline{X}) = 1.1692,$ $Z_2(\overline{X}) = 2.1461.$ Also achieved values of membership functions are: $\mu_1(Z_1(X)) = 0.99999,$ $\mu_2(Z_2(X)) = 0.99994.$

Comparison table (table 1) and comparative graph (bar chart 2) between proposed revised FGP approach and FGP approach given by Lachhwani [2] show that the both the approaches are close to one another but the modified methodology requires less computational work in terms of constructing separate membership functions for numerator and denominator objective functions.

For numerical example 1 Revised FGP approach	FGP approach byLachhwani [2]	Optimal solution
$\mu_{z_1}(N_1(\mathrm{X})) = 0.8333,$ $\mu_{z_2}(N_2(\mathrm{X})) = 0.8331,$ $\mu_{z_1}(D_1(\mathrm{X})) = 1,$ $\mu_{z_2}(D_2(\mathrm{X})) = 1.$	$\mu_1(Z_1(X)) = 0.99999,$ $\mu_2(Z_2(X)) = 0.99994.$	$Z_1(\overline{X}) = 1.1692,$ $Z_2(\overline{X}) = 2.1461.$
at $X(x_1, x_2) = (1.3333, 0)$	at $X(x_1, x_2) = (1.3333, 0)$	

Table 1. Comparison of membership function values for example 1



Bar chart 2. Comparison of Membership function values with two FGP approaches (For example 1)

5. Conclusion

This paper presents modified procedure for MOLFP based on fuzzy goal programming approach which yields better compromise optimal solution of problem with a higher degree of satisfaction. Also, the proposed technique is efficient and requires less computational work.

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